Reliability-based approach to the assessment of hydrate formation probability in deep-sea wet-gas pipelines

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Abstract: Comprehensively considering operation parameters, fluid physical parameters and the uncertainty of environment parameters, this work investigates hydrate formation probability in deep-sea wet-gas pipeline and proposes an efficient and accurate calculation method to predict hydrate formation probability in deep-sea wet-gas pipeline based on limit state equation, reliability theory, LHS (Latin Hypercube Sampling Method) - Monte Carlo sampling algorithm and Proper Orthogonal Decomposing method(POD). The stochastic numerical simulation method for different distribution functions is proposed on the basis of LHS - Monte Carlo algorithm. The sampling simulation of different random parameters at inlet of the pipeline under different distribution functions is performed by giving the suitable distribution function and parameter, and the influence of uncertainty factors on the calculation of hydrate formation probability is analyzed. Based on the POD algorithm, the use of simple interpolation calculation helps reconstruct the physical field. The reconstruction improves the problems of Monte Carlo algorithm, like large amount of computation and time consuming. Therefore, it increases the computational efficiency of the numerical simulation.

Keywords: Flow Assurance; Hydrate Formation; Latin Hypercube Sampling; Monte Carlo; Proper Orthogonal Decomposing.

1 INTRODUCTION

During the development process of deep-sea oil and gas, the safe operation of the deep-sea wet-gas pipeline has become increasingly prominent. It is always main issues that oil companies and scholars concern, issues including how to ensure the safe operation of the deep-sea wet-gas pipeline with low temperature and high pressure, and how to avoid hydrate formation and its blockage risk[1]. At present, there are mainly two methods to control hydrate formation and its blockage risk: the traditional inhibition method and the risk control method[2]. Currently, the traditional inhibition control method has been used mainly. But due to its drawbacks, like technical limitation and high cost, many researchers have started focusing on the risk control method. The development status of risk control method for hydrate is currently in experimental and theoretical studies, and it is experiencing testing process. Do research on hydrate formation mechanism and its flow characteristics within the wet-gas pipeline flow system in the deep-sea, and then do simulation and prediction the probability of hydrate formation. The research will help provide strong theory for the control technique of hydrate.

The large number of parameters that are greatly related to the formation process of hydrate is of significant uncertainty[3]. Currently, researchers in the fields, like aerospace and structural engineering, have widely used reliability-based design and evaluation method to study the uncertainty of these parameters[4]. The reliability-based design and evaluation method is a probabilistic design method. The core idea is to comprehensively evaluate the effect of each various random variable on the system security, and to quantify the risk characterization using the system failure probability at a particular limit state.

This paper will be based on the analysis method of piping structural strength reliability, and the paper will establish the ultimate limit state equation based on reliability theory. The equation is for the prediction of the conditions of hydrate formation. Use Latin Hypercube sampling (LHS), Monte Carlo algorithm, and Proper Orthogonal Decomposition (POD) calculation, to help create the initial sample matrix of random variables' sample geometric and physical field. Then use numerical simulation to predict the probability of hydrate formation within gas pipeline under the influence of different variables. The prediction will achieve the risk assessment of hydrate formation in deep-sea wet-gas pipeline. Based on the risk assessment, the paper will develop effective prevention measures and control measures, which will help reduce economic losses as low as possible. The research will provide new theoretical support for the development and transportation of deep-sea oil and gas.

2 THE ULTIMATE LIMIT STATE EQUATION OF HYDRATE

Ultimate Limit State is a critical state, and after exceeding the limit state, the system could not meet any functional requirement of design settings. Normally, the limit state of the pipeline includes two categories: one limit state is dependent on the time, such as corrosion of pipeline, and its reliability varies over time; the other limit state is independent of the time, such as hydrate formation within the pipeline under steady state flow condition, and its reliability is not altered over time.

To study the hydrate formation in the wet pipeline, the paper studies the difference between the actual temperature of the pipeline and the temperature at which hydrate forms in the pipeline. Then use the temperature difference to establish probability limit state equation for hydrate formation. The temperature and the pressure to form the gas hydrate are the corresponding. Under the same pressure condition, compare the gas flow temperature within the pipeline with the temperature where the hydrate forms at this pressure. The comparison can determine whether there is hydrate formation in the pipeline. Therefore, this article builds the limit state equation for hydrate formation in the gas pipeline, which is shown as formula (2.1) or (2.2).

$$W(x_i) = T_{Hyd,p_i} - T_i \qquad (2.1)$$

$$W(x_i) = p_i - p_{Hyd,T_i} \qquad (2.2)$$

If $W(x_i) > 0$, it means the *i* node location falls into the region where hydrate forms; if $W(x_i) < 0$, it means the *i* node location do not fall into the region; if $W(x_i) = 0$, it means the *i* node location is on the curve where indicates hydrate forms.

In addition, there is a necessary condition for hydrate formation, which is that there must be the presence of free water within the pipe. Therefore, the equation mentioned above needs to be modified, shown as formula (2.3).

$$Z(x_i) = W(x_i) \bullet w(x_i) \qquad (2.3)$$

If $w(x_i) = 1$, it indicates the presence of free water at the *i* node location; and if $w(x_i) = 0$, it indicates the absence of free water at the *i* node location.

$Z(x_i)$, reference value of hydrate formation, which determine whether there is hydrate forming or not at i^{th} node location along the pipeline. If $Z(x_i) > 0$, it indicates hydrate forms, and if $Z(x_i) < 0$, it means hydrate will not form.

3 THE RANDOM VARIABLE SIMULATION ALGORITHM

The core of computing the probability of hydrate formation is to calculate the probability of its failure. The calculation of the probability of failure includes the following three methods: analytical method, embedded method, and Monte-Carlo algorithm. Analytical method is to transfer the random variables into the normal distribution variables mutually independent in standard probability space, and then simplify and approximate the boundary of integration region to be a probability integral. The advantages of Analytical method are high speed and accurate results, but it need to know the analytic function of the studied issues, and the analytic function must be the first order derivative[5]. Embedded method (such as the response surface method[6]) firstly need to determine the failure boundary. Monte-Carlo method, also known as random simulation method or statistical testing method, is the powerful tool to study the complex system of multi-variables and their uncertainties. When the sampling frequency is sufficient for a long time, it can be considered as a result with the exact solution[7]. It need to be noted that, despite that the analytical method and embedded computing method have the advantages of fast calculation and accurate results, they are only applied to simple question. For the complicated question like the probability of hydrate formation, only Monte-Carlo method can be used.

3.1 LHS-Monte Carlo random sampling algorithm

Monte Carlo algorithm, also known as computer statistical simulation method, is a numerical calculation method based on the statistical probability theory, and solve computational problems using the random numbers (commonly pseudo-random numbers). Monte Carlo algorithm has been widely used in the fields like finance, economics, and physics.

Monte Carlo algorithm is based on the Bernoulli law of great numbers of the statistics probability theory. Its convergence divergence depends on the number of random parameters. The more sampling times, the more accurate the result. It has the drawbacks of low sampling efficiency and samples covering heterogeneity. For general simple questions, the sampling frequency can be 10^4 to 10^6 times, but for complex problems, it is required a very large number of analog sampling. In recent years, scholars have proposed to reduce the variance to reduce the number of simulations, which is to improve simulation efficiency. For specific questions, scholars have proposed different methods to reduce variance, such as importance sampling, systematic sampling, and stratified sampling, etc. This paper select a stratified sampling -- Latin Hypercube Sampling, referred to as the LHS.

LHS algorithm is a sampling method of small samples, and it has obvious advantages to simulate the event with small probability. The core idea is that after *M* times of sampling of the specified random variables, divide the range of 0-1 into *M* non-overlapping contour intervals, then randomly pick a value in each contour interval. Thus it helps ensure that the probability of each value is 1/M. Do calculation using the medium value of each contour interval as a random number, or using the result from formula (3.1).

$$U_i = \frac{V}{M} + \frac{i-1}{M} \tag{3.1}$$

Where, i = 1 ~ *M*, Obviously, for any one of the contour intervals, there is only one random number:

$$\frac{i-1}{M} < U_i < \frac{i}{M} \tag{3.2}$$

There are four methods to generate non-uniformly distributed random numbers: Inverse Transform Method, Acceptance Rejection Method, the composite method, changes and look-up table method. For actual application, it need to combine with the specific distribution function and then select proper generation method. The paper studies the probability of hydrate formation within gas pipelines using LHS-Monte Carlo sampling algorithm. Use LHS-Monte Carlo sampling algorithm to do randomly sampling of pressure, temperature and flow rate at pipeline inlet. Then calculate the random numbers with non-uniformly distribution using Inverse Transform Method.

Suppose the probability distribution function of the random variable *X* is *F(X)*, make *U=F(X)*, and then apparently *U* obey uniform distribution with the probability within *0-1* range. Therefore, a random number of the random variable X generated by the Inverse Transform Method can be mainly reached by two steps:(1) generating a series of random numbers uniformly distributed within the 0-1 range; (2) calculating the random variable of distribution function *F(X)* based on the function $X = F^{-1}(U)$, and the distribution function *F(X)* must satisfy the cumulative probability distribution. Table 3.1 shows several common calculation methods of distribution generation.

Table3.1 Generating algorithm of common probability distribution

Distribution Type	Probability Distribution Function	Inverse Transformation Function	Random Number
Uniform Distribution	$U=\begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b<x \end{cases}$	$F^{-1}(x) = a + (b-a)u$	$X_i = a + (b-a)u_i$
Normal Distribution	$U = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{(s-\mu)^2}{2\sigma^2}\right] ds$	$y = \sqrt{-2\ln u}\ \sin 2\pi u$	$X_i=\mu+\sigma y_i$
Lognormal Distribution	$U = \int_0^x \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{(\ln t-\mu)^2}{2\sigma^2}}$	Obtain X based on the generation method of random matrix of normal distribution	$Y_i = e^{X_i}$

Exponential Distribution	$U=\begin{cases}1-e^{-\lambda(x-\mu)} & (x\ge\mu)\\ 0 & (x<\mu)\end{cases}$	$F^{-1}(x)=\mu-\frac{\ln(1-u)}{\lambda}$	$X_i=\mu-\frac{\ln(1-u_i)}{\lambda}$
Gumbel Distribution	$U=\exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}$	$F^{-1}(x)=\mu-\sigma\ln(-\ln u)$	$X_i=\mu-\sigma\ln(-\ln u_i)$
Rayleigh Distribution	$U=\begin{cases}1-e^{-\frac{x^2}{2\mu^2}} & (x\ge 0)\\ 0 & (x<0)\end{cases}$	$F^{-1}(x)=\sqrt{-2\mu^2\ln(1-u_i)}$	$X_i=\sqrt{-2\mu^2\ln(1-u_i)}$

During the state flow process of gas-water two phases, along with the fluctuations of the pressure, temperature and flow rate at pipeline inlet, the paper mainly considers the prediction of probability to form hydrate along each node location of the pipeline. Through the statistics and analysis of existing experimental data and field data, the paper generates preliminarily the probability distribution function of all parameters at pipeline inlet. However, due to no finding of universal probability distribution model for parameters at inlet, the paper believes that the random variables of inlet would satisfy one of these five distribution models (normal, lognormal, exponential, Gumbel, and Rayleigh). Therefore, apply the analysis of domain of definition to five distribution models mentioned above, use Monte Carlo algorithm based on LHS stratified sampling, and achieve the random sampling for inlet parameters which satisfies a certain distribution features.

According to Table 3.1, the paper takes the inlet pressure as example to illustrate the random variable's sample value generated by LHS-Monte Carlo simulation under different distribution functions. The result of inlet pressure is shown on table 3.2. Table 3.2 shows the inlet pressure distribution under different distribution models. Normal distribution result is shown as Figure 3.1, and the rest of distribution is shown as Figure 3.2.

Table 3.2 Check list of distributed parameters of inlet pressure

Distribution Type	Mean	Standard Deviation	Coefficient of variation	Number of sampling
Normal Distribution	6	0.8	0.133	1000
Lognormal Distribution	7	3.7	0.533	10000
Exponential Distribution	8	8	1.0	10000
Gumbel Distribution	8.28	2.5	0.310	10000
Rayleigh Distribution	7.05	3.7	0.523	100000

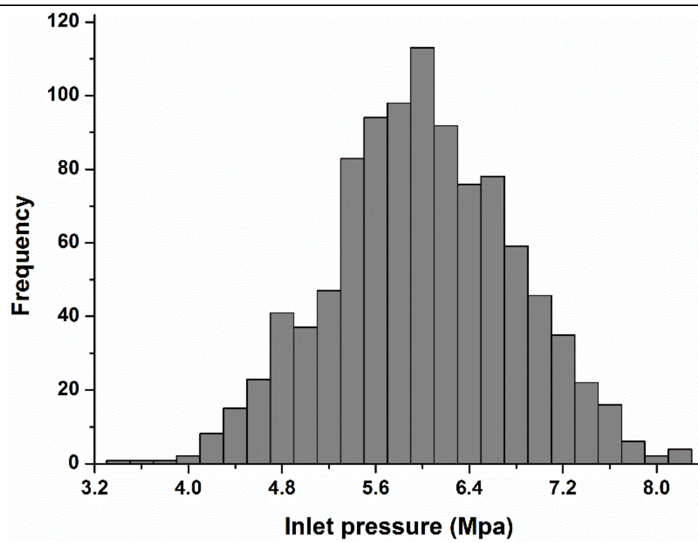

Fig. 3.1 The normal distribution histogram of inlet pressure

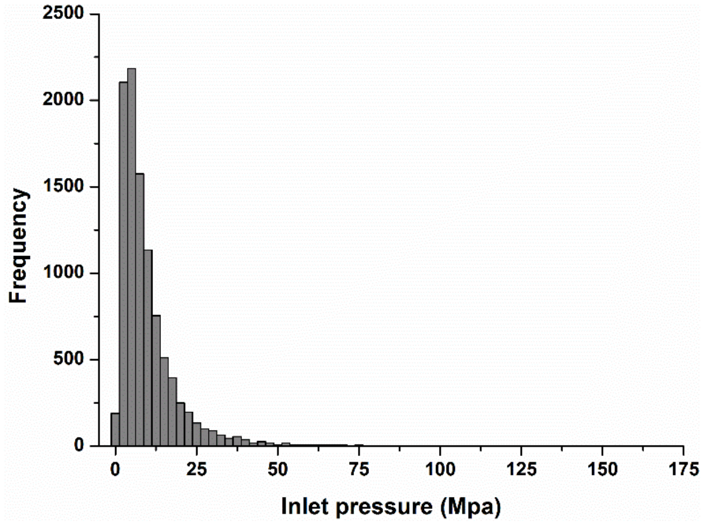

Fig. 3.2 The lognormal distribution histogram of inlet pressure

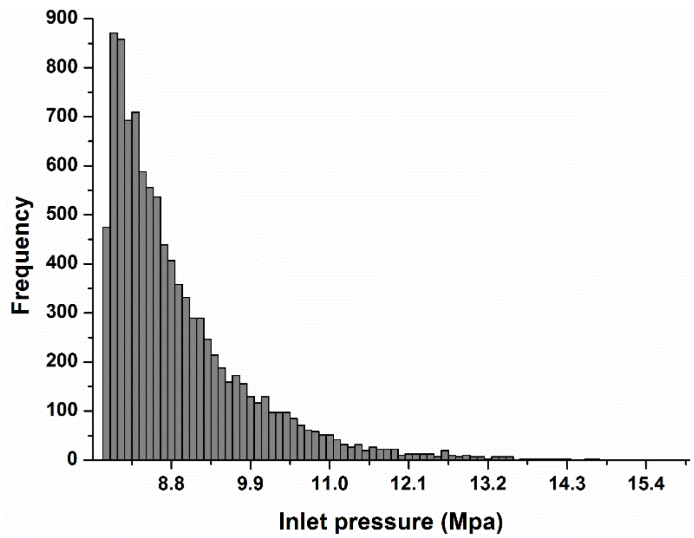

Fig. 3.3 The exponential distribution histogram of inlet pressure

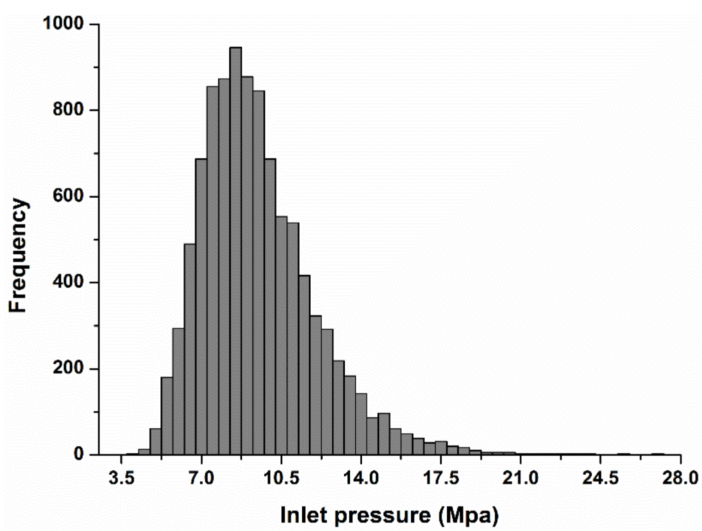

Fig. 3.4 The Gumbel distribution histogram of inlet pressure

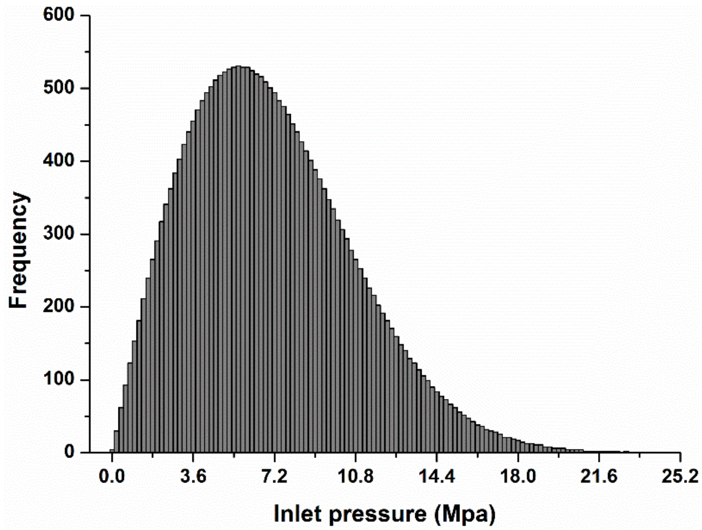

Fig. 3.5 The Rayleigh distribution histogram of inlet pressure

3.2 POD algorithms and its application

When system variables become random variables, Monte-Carlo algorithm is commonly used to carry out random sampling for random variables, to calculate the limit state of physics problems, and to compute the failure probability of research objects[8]. When using Monte-Carlo random sampling to calculate the failure probability, normally, the simulation process need to increase the sampling frequency, which is to ensure the convergence and the accuracy of the simulation results. The increasing sampling frequency increases the amount of computation. This paper introduces POD (Proper Orthogonal Decomposition, abbreviated as POD) algorithm. Among the random variable samples from Monte-Carlo algorithm, select a small amount of samples to do numerical calculation, which constitutes the initial sample matrix. Using POD to do orthogonal decomposition of the initial matrix can achieve the basic function library which could describe the physical problem. The rest samples unselected can be reached by simple algebra calculation. The POD avoids the duplicate calculation of such large amount of samples during the numerical simulation process, which is to achieve fast simulation calculation with high accuracy.

Implementation steps of POD algorithm consist of three main parts. The first step is to generate the basic sample space, and ensure the basic samples' coverage as uniform as possible. The second step is to decompose the singular values of the sample space, and to obtain eigenvalues and matrix functions. The third step is to calculate the corresponding general coefficients according to the basic functions and the sample space, and to reconstruct physical field through simple calculation of the basic functions and the general coefficients. The programming involved in this section is achieved via Visual Studio 2010 C ++ programming software.

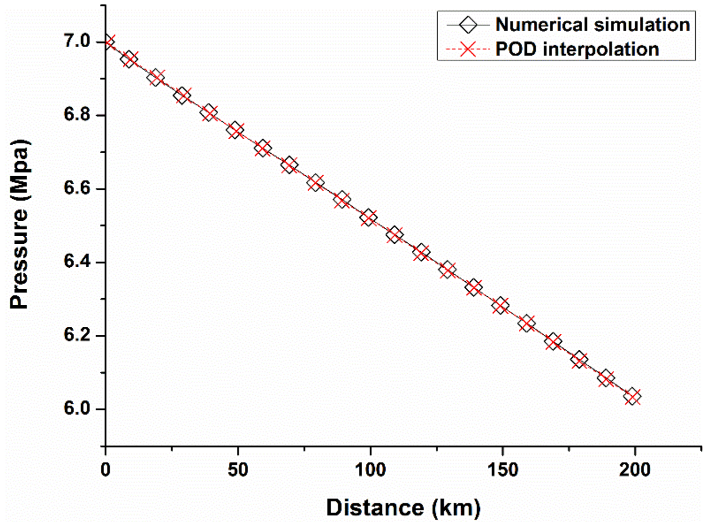

Fig. 3.6 The comparision of POD interpolation and numerical simulation

4 THE EVALUATION OF THE PROBABILITY OF HYDRATE FORMATION

Based on the reliability theory and quantitative risk assessment methods, the paper completes the research to estimate the probability of hydrate formation in gas pipeline, through the idea of reliability analysis of pipeline structure. Specific research ideas is shown in Figure 4.1.

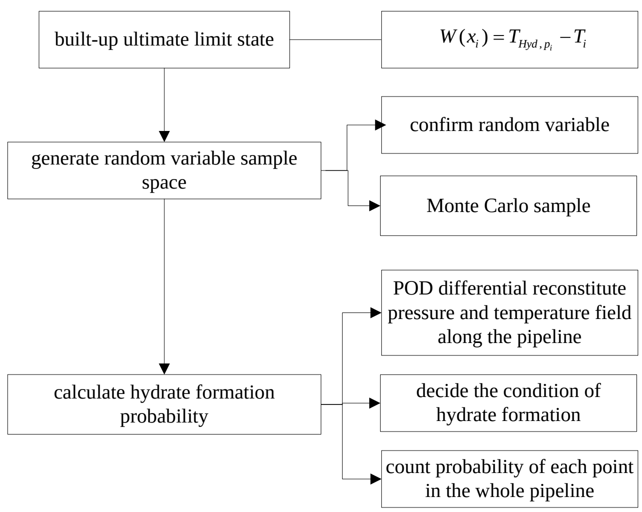

Fig. 4.1 The idea of calculating probability of natural gas pipeline hydrate formation

The Figure 4.2 describes the process diagram to calculate the formation probability of hydrate in gas pipeline using POD-Monte Carlo algorithm. The program uses Monte-Carlo random sampling method which is the optimization process of LHS algorithm. Based on the Bernoulli law of large numbers, the accuracy of Monte-Carlo algorithm increases along with the increasing number of samples during the process of probability estimation. It also comes with a large number of duplicate calculation. POD algorithm helps compress the storage space through the basic functions and general coefficients achieved by calculation, and reconstructs the physics space by interpolation. Then, the use of POD algorithm will shorten calculation time.

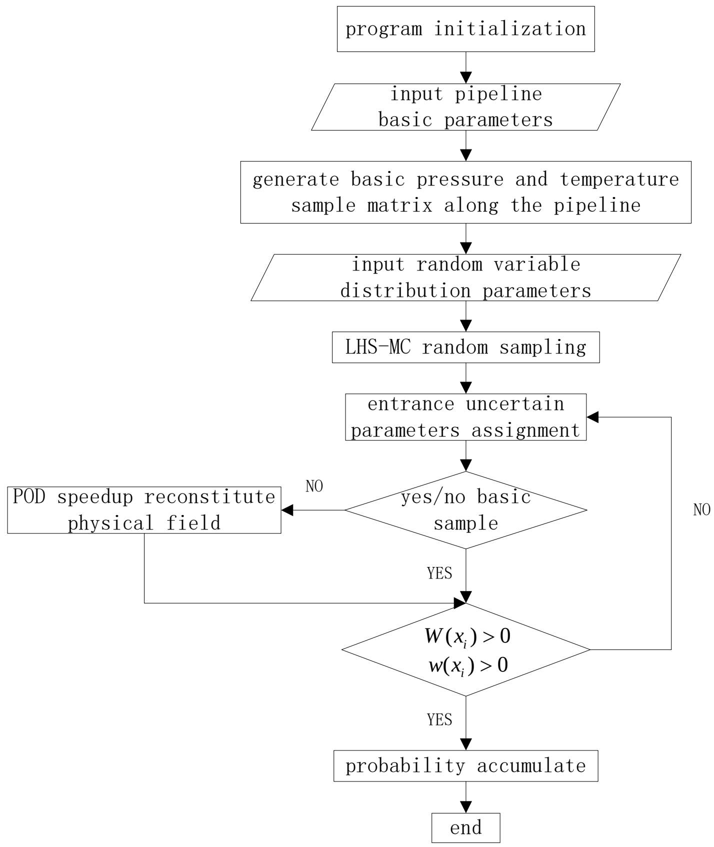

Fig. 4.2 the process diagram to calculate the formation probability of hydrate in gas pipeline

5 EXAMPLE CALCULATION AND RESULTS ANALYSIS

The total length of a long-distance gas pipeline is 100km with specification of Φ660×7mm. Its wall roughness is 0.015mm, and it is buried deep in the ground where the temperature is 7℃. The overall heat transfer coefficient is 1.5 W/(m^2•K), the inlet pressure is 6MPa, the inlet temperature is 30℃, and the flow rate is 67.6 kg/s. The natural gas composition is shown in Table 5.1, and the mileage-elevation distribution along the pipeline is shown in Figure 5.1. Also, the moisture content of gas in the pipe inlet is 300mg/m^3 (20 ℃, 0.101325MPa).

Table 5.1 The natural gas composition for a pipeline (mol %)

composition	C_1	C_2	C_3	nC_4	N_2	CO_2	H_2S	Total
The molar composition	90.52	6.249	0.45	0.08	1.57	1.13	0.001	100

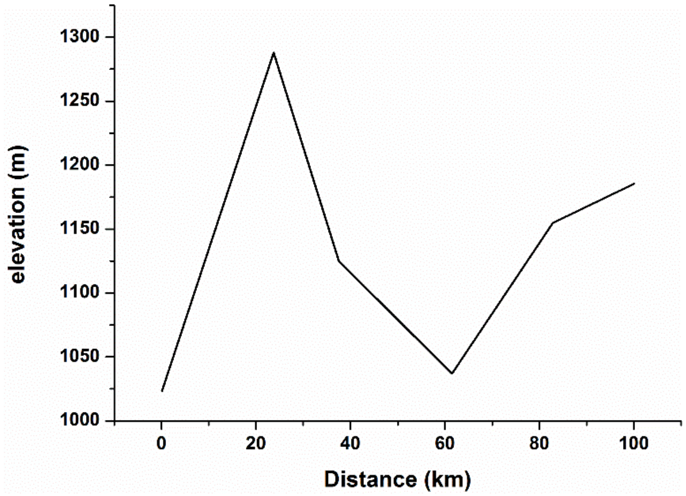

Fig. 5.1 The mileage-elevation distribution along the pipeline

5.1 Selection of random inlet variables

Use the mean value of each distribution model shown in table 3.1 and the standard deviation formula to calculate the probability distribution parameters of inlet random variables of this case, and the result is shown in table 5.2. Where in table 5.2, inlet values of p, T, and Q have the same mean values and different standard deviation under respective five different distribution models.

Table 5.2 Distribution models and model parameters of the random variables

Random parameters	Normal distribution	Lognormal distribution	Gumbel distribution	Exponential distribution	Rayleigh distribution
Inlet pressure, p（Mpa）	N(6.0,0.9^2)	Ln(1.76,0.25^2)	G(4.6,3.0)	E(5.6)	R(4.79)
Inlet temperature, T（℃）	N(30.0,1^2)	Ln(3.37,0.10^2)	G(29.3,1.5)	E(29.0)	R(23.94)
Inlet flow rate, Q（kg/s）	N(67.6,1^2)	Ln(4.18,0.25^2)	G(66.7,2.0)	E(66.0)	R(53.94)

The following sections take samples of the normal sample as main consideration, and are supplemented by samples of various other sample distribution models. Then, analyze the probability of hydrate formation in the pipeline under different inlet parameters' samples.

5.2 The effect of one single random variable on the probability of hydrate formation

Calculate the probability of hydrate formation within gas pipeline using the random variable of inlet pressure, inlet temperature, and inlet flow rate, respectively. Use LHS-Monte Carlo to randomly select 100 samples of inlet pressure, inlet temperature and inlet flow sample value, respectively, and values are subjected to the normal distribution in table 5.2). Then, use POD interpolation to reconstruct the pressure field and temperature field of the pipeline. Finally, calculate the probability of hydrate formation at each node location along whole pipeline. Compare the probability result mentioned above with that of the inlet fixed parameters (inlet pressure of 6MPa, inlet temperature of 30 ℃, and inlet flow of 67.6kg/s).The result is shown in Figure 5.2 which only provide the date from 50th location to 102nd location, and Figure 5.2 shows that the probability of hydrate formation at and before 57th location is zero. Compare four lines in Figure 5.2, and it can be seen that along the pipeline, the number of locations where hydrate forms and the probability values of hydrate formation change with the random change of inlet parameters. Different random variables have different degrees of influence on the results. Compare the probability curves with and without the effect of random variables, and the influences of the inlet parameters of the pipeline as uncertainty factors on the hydrate formation probability was ranked as P>T>Q.

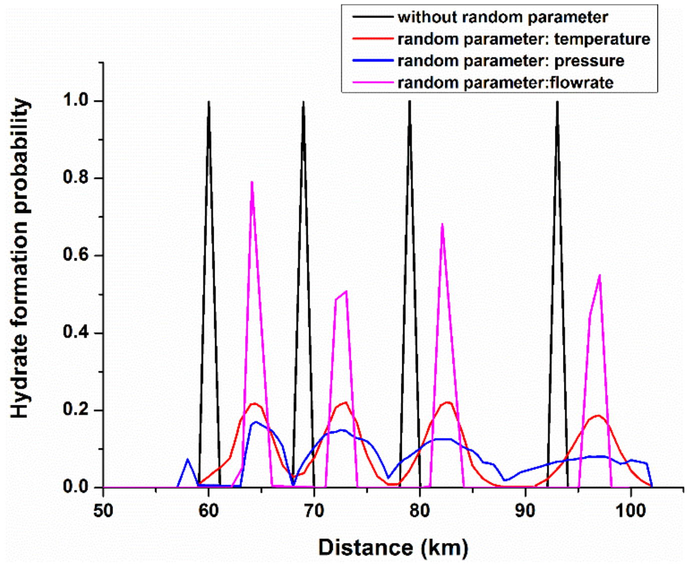

Fig. 5.2 Influence of random variables on hydrate formation probability

5.3 The effect of the number of random variables on the probability of hydrate formation

Figure 5.3 shows the distribution of the probability of gas hydrate formation along the pipeline under the situations with one, two and three random variables (situations satisfy the normal distribution function in Table 5.2). In order to ensure uniformity of the samples of random variables, the analysis process use 10000 sets of each inlet parameter (P, T, and Q). According to Figure 5.3, the probability curve's volatility is greater when the inlet temperature is random variable. The probability curve of inlet pressure as the random variable is similar to the probability curve with double parameters' effect. Inlet parameters under different conditions will affect the probability of hydrate formation in the pipeline, and the degree of effect varies.

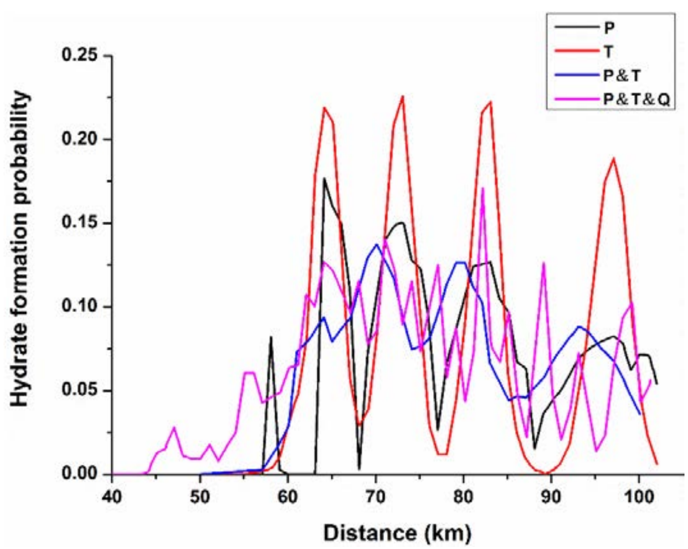

Fig. 5.3 Influence of Normal distribution sampling of different inlet parameter on hydrate formation probability

5.4 The effect of the number of a single random variable on the probability of hydrate formation

Figures 5.4 to 5.6 present the effect of different number of samples on the probability of hydrate formation under the situations of the inlet pressure, temperature, and flow rate satisfying the normal distribution. Figure 5.4 demonstrates that when the number of inlet pressure samples reaches 100 and above, the curves begin stable. But curves still have the fluctuation within a certain range. Figure 5.5 shows that when the number of the inlet temperature samples reaches 100 and above, the curves begin stable. And Figure 5.6 shows that the change in the number of the inlet flow rate samples has little effect on the probability of hydrate formation. In summary, it is believed that when the number of random sampling of random variables reaches 100 and above, the influence of the number of random samples on the probability of hydrate formation in gas pipeline becomes stable.

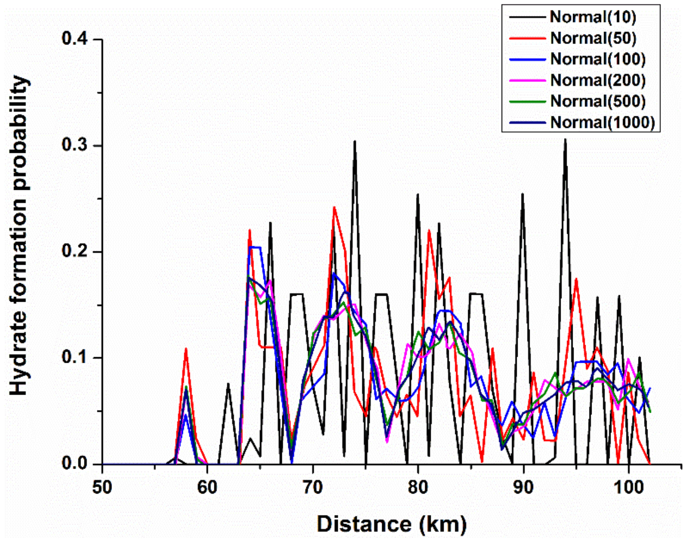

Fig.5.4 Influence of different number of Normal distribution sampling of inlet pressure on hydrate formation probability

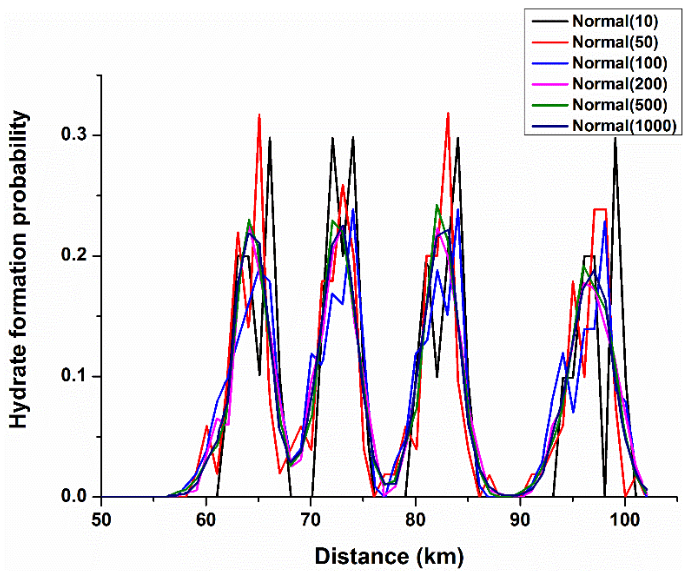

Fig. 5.5 Influence of different number of Normal distribution sampling of inlet temperature on hydrate formation probability

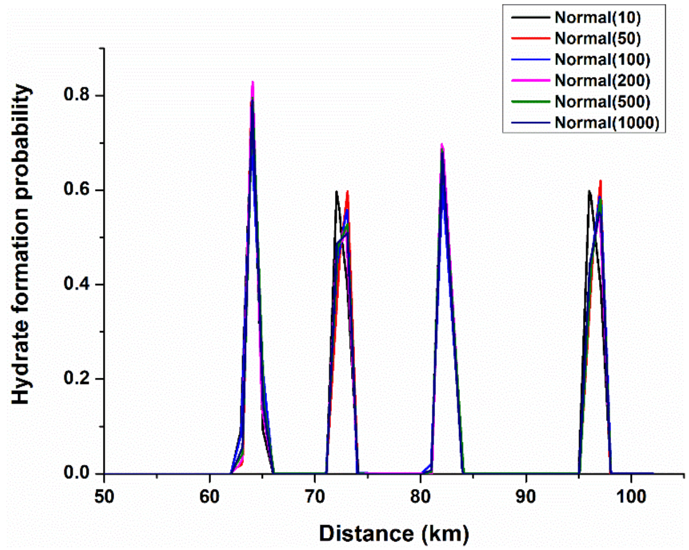

Fig. 5.6 Influence of different number of Normal distribution sampling of inlet flow rate on hydrate formation probability

5.5 The effect of different distribution samples of a single random variable on the probability of hydrate formation

Figures 5.7 to 5.9 demonstrate the effect of different random distribution samples of inlet pressure, temperature and flow rate on the probability of hydrate formation. According to the five random distribution functions listed in Table 5.2, select 100 samples of random variables of each distribution, then do statistics to get the distribution of the probability of hydrate formation under different random samples.

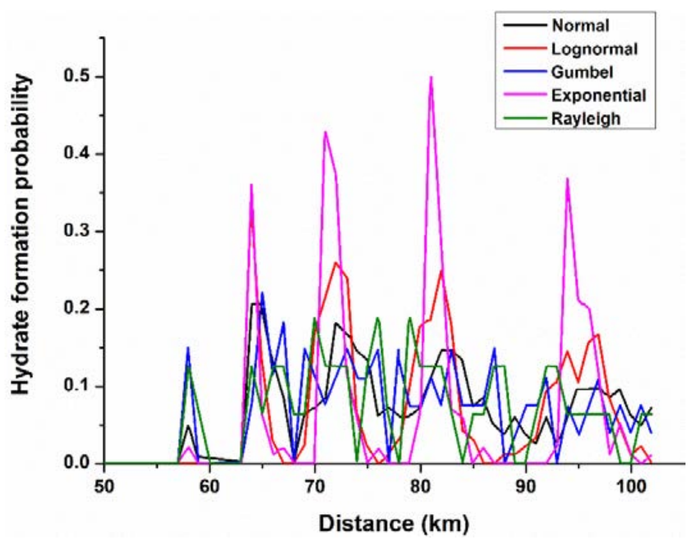

Fig.5.7 Influence of different distribution sampling of inlet pressure on hydrate formation probability

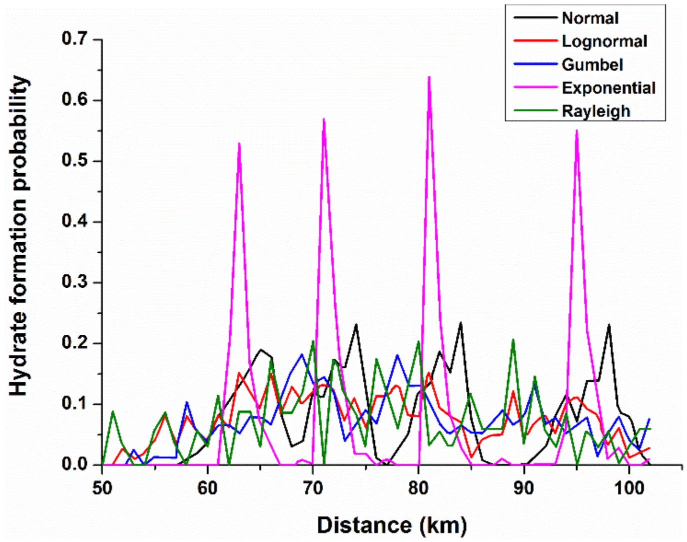

Fig.5.8 Influence of different distribution sampling of inlet temperature on hydrate formation probability

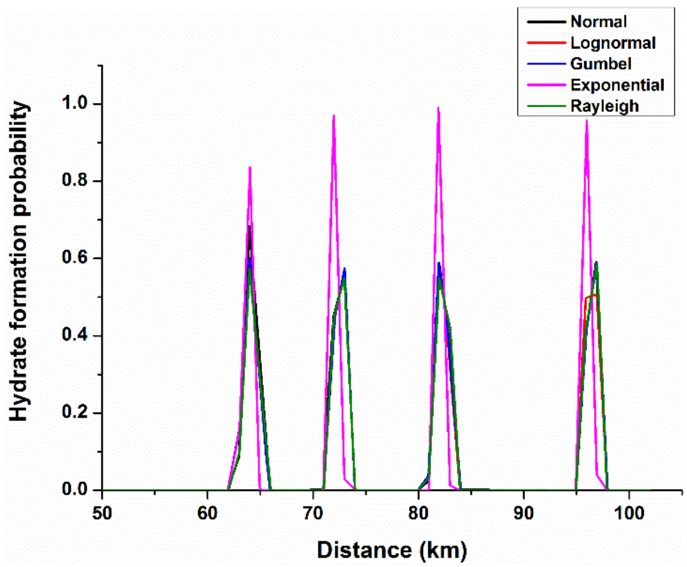

Fig.5.9 Influence of different distribution sampling of inlet flow rate on hydrate formation probability

Figures also present that the inlet values (P, T, Q) under the condition of the exponential distribution have the greatest effect on the probability of hydrate formation. The probability curve with the effect of the exponential distribution is obviously greater than that of other distribution models. According to Figure 5.7, with the effect of the lognormal distribution, the probability value with the effect of inlet pressure is relatively greater, and the probability curves with the effect of other three distribution models also have the fluctuation within a certain range, but the degree of fluctuation is much less. Figure 5.8 indicates that the probability value with the effect of inlet temperature under the normal distribution is greater as well. And with the effect of the random inlet temperature from the Rayleigh distribution, Gumbel distribution and the lognormal distribution, the probability curves of hydrate formation are distributed more flat, and the probability values are relatively small. Figure 5.9 shows that for the effect of inlet flow rate on the probability of hydrate formation, there is insignificant difference under the other four distribution models, except the exponential distribution.

6 CONCLUSIONS

The probability of hydrate formation in the wet gas pipeline is the base to evaluate the safe flow of wet gas pipeline. The paper analyzes the change of the probability of hydrate formation in the pipeline with the effect of inlet pressure, inlet temperature, inlet flow rate and their combined effect under different distribution models. Different random variables, difference probability distribution of random variables, and the different numbers of samples of random variables will all have the effect on the calculated probability values of hydrate formation in the pipeline, and the degree of effect are different. Reasonable determination of the distribution function and the number of samples of the random variables can effectively reduce the workload of simulation calculation. During the design and production processes, the proper selection or adjustment of inlet random parameters enables he rapid decline in the probability of hydrate formation, so as to provide the basis to ensure pipeline's safe operation.

7 ACKNOWLEDGEMENTS

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Nomenclature

x_i	[-]	the parameter at i^{th} node location along the pipeline
T_i	[℃]	the actual temperature at i^{th} node location along the pipeline
pi	[Mpa]	the actual pressure at i^{th} node location along the pipeline
$T_{Hyd,pi}$	[℃]	the temperature to form hydrate at pressure p_i at at i^{th} node location along the pipeline
$p_{Hyd,pi}$	[Mpa]	the pressure to form hydrate at temperature T_i at at i^{th} node location along the pipeline
$W(x_i)$	[℃/Mpa]	reference parameter
$w(x_i)$	[-]	the predicted value of the free water
$Z(x_i)$	[℃/Mpa]	Reference value of hydrate formation
V	[-]	represents a random number uniformly distributed on 0-1
U_i	[-]	represents the random number of the i^{th} interval
$(i-1)/M$	[-]	lower boundary of the i^{th} interval
i/M	[-]	upper boundary of the i^{th} interval
a	[-]	lower boundary of uniformly distributed
b	[-]	upper boundary of uniformly distributed
μ	[-]	mean
σ	[-]	standard deviation
λ	[-]	rate parameter