A Bayesian analysis of the risk of satellite collisions and of space surveillance improvements

M-Elisabeth Paté-Cornell* and Richard Kim*
Management Science and Engineering
Stanford University

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Abstract: As new spacecraft are inserted in various earth orbits, the risk of collisions with active or inactive spacecraft and debris increases. Currently, several space surveillance systems provide signals of impending collisions and opportunities for owners and operators to move a threatened satellite. These monitoring systems, however, are imperfect, and the current number of sensors of different types, costs, and levels of accuracy may need to be increased to allow better space traffic management. This paper presents a Bayesian method to estimate and optimize the benefits of such improvements, based on the concept of value of information to compare alternatives.

Keywords: Satellites, Collision risks, Warning systems, Monitoring, Value of information

1. INTRODUCTION

The world depends increasingly on the operation of satellites and their constellations (e.g., the Global Positioning System) and the number of satellites in orbit is steadily increasing. The focus of this paper is on the risk of unintended collisions of active satellites with active or inactive ones and with debris. As the number of satellites increases, so does the number of pieces of dead spacecraft. This collision risk is currently managed through monitoring systems that provide signals of collision threats. These signals allow the owners/users of satellites to take protective measures such as moving a satellite out of the way. Current ground-based sensor systems include both highly reliable but expensive units such as the US Space Surveillance Network (A system), and systems of less accurate but less expensive sensors operated by a number of companies† (B systems). The A system –which is here a schematic representation of the main surveillance system in the US- includes a number of sensor classes, each monitoring a set of satellite constellations. The B system is less structured and composed of sensors from various sources that monitor everything that they can see around the world and share that information with governments and industries. Although A is more accurate than B, both systems send generally reliable if imperfect signals, with probabilities of false positive and false negatives. It is assumed here that these probabilities of errors are the same for all sensors of class A and for all sensors of class B and that all signals are independent conditional on a collision threat. Any figure used here is purely illustrative and does not represent the accuracy of existing sensors.

The prior probability of a collision in space is very small and only a few collisions have occurred in the last decades. It is also very uncertain as the collision causes vary. Studies estimate that the steady-state probability of any active satellite colliding with another orbiting body in space is in the range of 10^{-8} to 10^{-4} per spacecraft and per year [1]. Yet, the continued proliferation of space debris, the accelerating rate of launches, thus the congestion of space especially in Low-Earth Orbit, is bound to increase considerably this collision risk.

* mep@stanford.edu, richhkim@stanford.edu
† AGI, for example, is a private company that fuses "satellite tracking measurements from a global network of commercial sensors" and provides monitoring results to both government and industry

The purpose of the Bayesian model presented further is to upgrade these priors when signals are observed, in order to support two decisions [2]. One is the tactical decision of the owner/operator of a satellite to move a threatened satellite out of the way. The other is the strategic decision of the managers of the global surveillance system to invest in additional sensors in one of the two systems. We consider here the choice between adding one sensor in the A system, or several sensors for the same price in the B system. The Bayesian model involves a probabilistic aggregation of a number of signals from both systems to assess the posterior probability of a collision given a threat message. For a constellation to fail, a minimum number of satellites have to fail. We compute the posterior probability that this number of failures is reached or exceeded at any time unit given that signals are received and evasive measures can be taken. We then use the classical concept of value of information to compare the risk reduction benefits of different options of monitoring improvements. We assume that the decision makers are not necessarily risk neutral, but have a constant risk aversion over the range of possible losses, hence an exponential "disutility" function for losses of data and costs.

2. SOME BACKGROUND

2.1 Satellites, monitoring and collisions

The value of satellites resides in the value of the data that they gather and send to a ground-based processor. Figure 1 represents a systems-level overview of satellite systems.

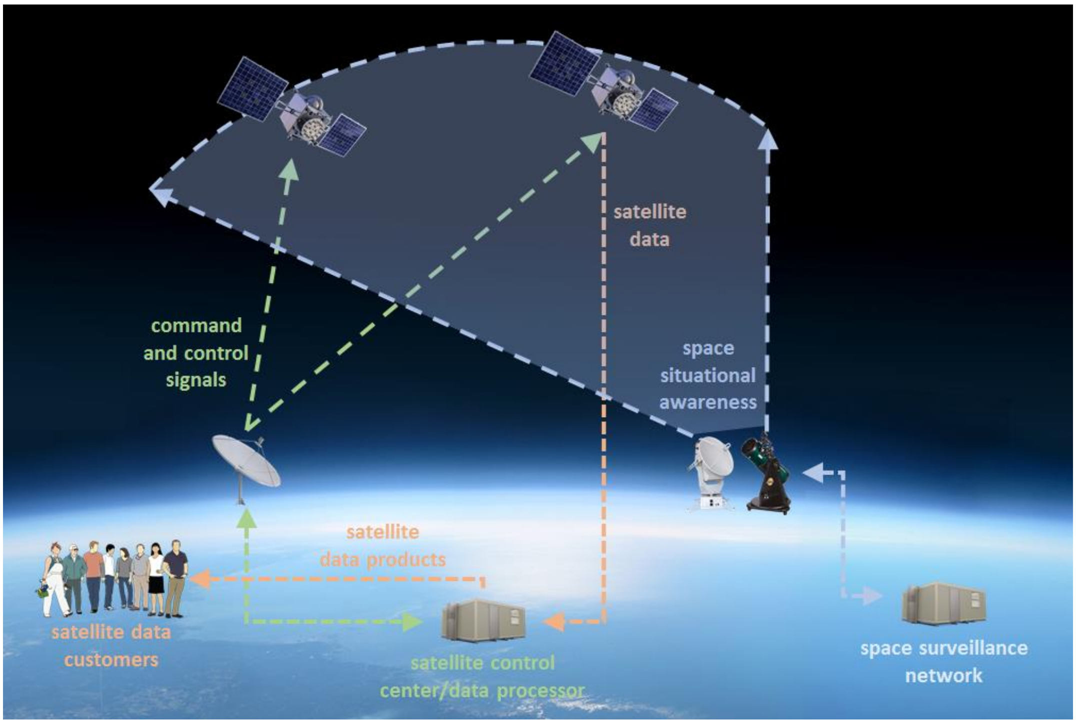

Figure 1. System-level overview of satellite systems, depicting satellite data signals routing to customers, command and control signals, and space surveillance [2]

The prior probability of a collision (in the absence of positive signals) depends on the orbit and the sensors that monitor constellations of satellites in that orbit. Orbits are grouped in several broad classes according to their altitude and eccentricity. The most populated orbits are Low Earth Orbit, Medium Earth Orbit, Geosynchronous Earth Orbit, and Highly Elliptical orbit (HEO) (see Figure 2). Our focus is mostly on the Low Earth Orbit since it involves the largest number of satellites and the highest density of debris.

The US Space Surveillance Network (system A) is essentially composed of a set of radars as represented in Figure 3. The number of spacecraft (active or inactive) tracked by that system has increased from one in 1957 -with the launch of Sputnik- to over 4,000 by 2016. The total number of objects (including satellites and debris particles) tracked by that network was over 17,000 as of 2016 [3]. It has increased markedly as the result of two events. On January 11, 2007, the Chinese military launched an experimental anti-satellite missile from the Xichang Satellite Launch Center. The missile's payload, a kinetic kill vehicle, collided with one of China's own weather satellites, Fengyun-1C, at an altitude of 865 kilometres. While the intent of this experiment was to demonstrate China's anti-satellite capabilities, it resulted in a debris cloud of hundreds of thousands of objects, and increased markedly the risk of satellite operations in low Earth orbit. Then on February 10, 2009, an inactive Russian communications satellite –Kosmos-2251– collided with an Iridium Communications satellite – Iridium 33 – at an altitude of 789 kilometres above the Taymyr Peninsula in Siberia. NASA estimates that the resulting debris cloud includes at least 1,000 pieces of debris greater than 10 centimetres [4].

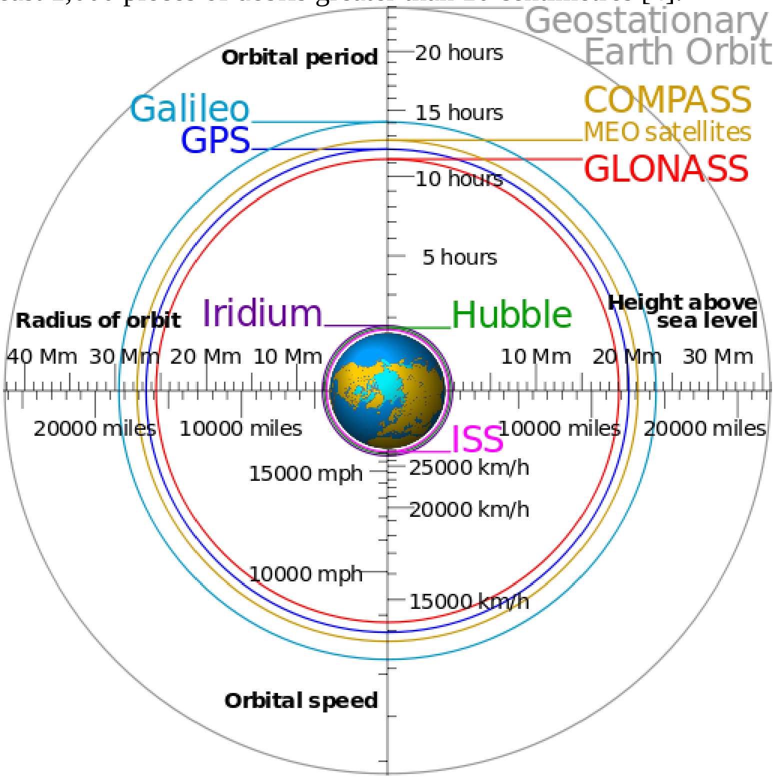

Figure 2. Summary of Earth orbits flattened to two dimensions, with selected examples of space systems and their orbits. Image from [5].

The US Space Surveillance Network provides satellite collision warnings worldwide from its different sensor groups. The process begins with the collection and aggregation of data from the network of sensors. These data are then synthesized at the Joint Space Operations Center where the information is correlated to an existing catalogue of tracked space objects. That catalogue contains about 30,000 entries, which represent the latest known number of known space objects. The catalogue is then used to anticipate the orbits of each of them, and to forecast potential collisions in future time windows (conjunction analysis). The results, however, are uncertain since ground-based optical sensors are subject to the variability of weather outages and some debris may be too small to observe yet potentially damaging. Collision predictions are also affected by the uncertainties of estimated orbits. If they are not updated regularly they are represented by large error ellipsoids to describe positional uncertainties. Furthermore, satellites illumination conditions can make their observation difficult. If the ground sensor line-of-sight into space lies along the leading edge of a satellite's solar panel, or if a satellite is in shade from the sun, even large satellites may be undetected.

Private companies also provide aggregated surveillance information to complement the major US network. At a fraction of the cost, for example AGI, which maintains an interactive space catalogues (the Next Generation Space Catalog) and since 2014, operates a Commercial Space Operation Center.

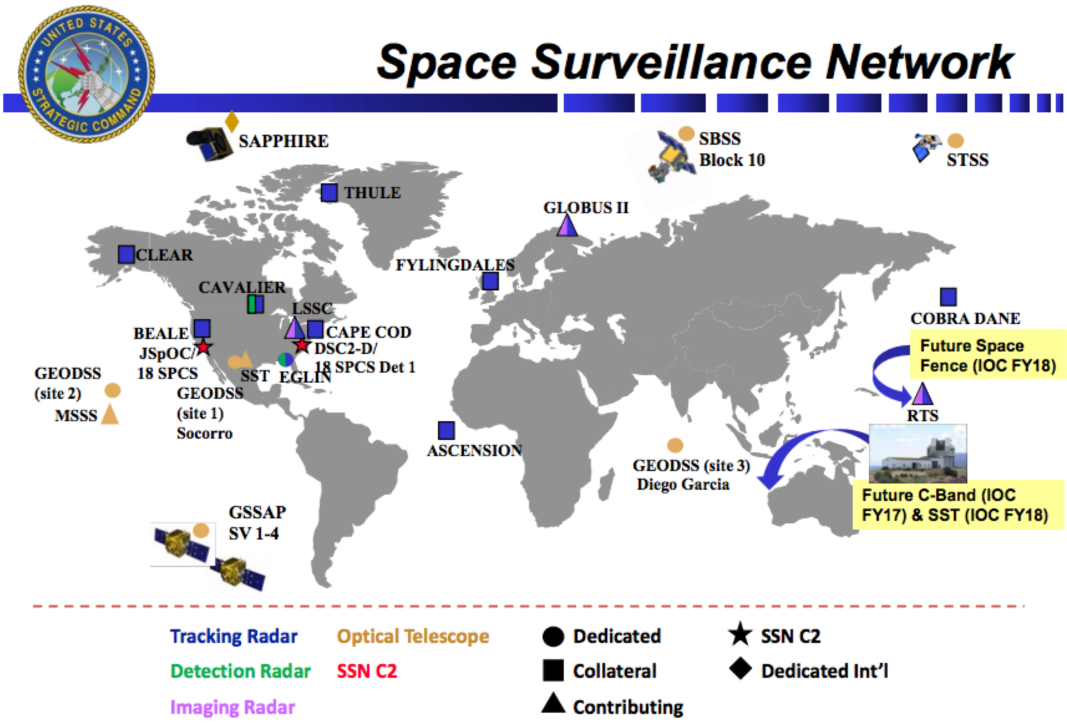

Figure 3: Overview of the US Space Surveillance Network. Source: [6].

2.2 Analytical methods: Bayesian updating, loss estimation and value of information

Decision analysis is the basis for the assessment of the risk reduction value of the signals from the monitoring system [7]. The first question is: what is the new probability of a collision after an observation of signals? Given the probability of a false positive or more likely, of false negatives, a "message" including one or more signals from either or both the A and B sensor systems, needs to be interpreted. The prior probability of collision is thus recomputed to inform the decision of whether or not to move the satellite out of the way. Bayesian updating is used to compute the posterior probability of a collision given a global message $\{m\}=\{n_{Aik}$ and $n_{Bik}\}$, which includes numbers of signals n_{Aik} from A and n_{Bik} from B regarding satellite k from constellation i. That posterior is the following:

p(collision|{m})=prior p(collision) x likelihood p({m}|collision threat)/p({m}).

These probabilities, and in particular the "pre-posterior" (probability of the message *a priori*) and the posterior (probability of a collision based on a message but before an evasive maneuver) are developed further in section 3.

The risk measure involves not only the collision probability but also the associated losses, which depend on the type of data that the satellites and/or the constellation generate, and for a specified decision maker, his/her risk aversion for these losses. In a 2014 report, the Satellite Industry Association estimated that the total direct revenue of the global satellite industry was $195.2 Billion in the fiscal year 2013 [8]. This figure includes direct satellite revenues for manufacturers, launch providers, ground station developers, and software providers. In addition, the loss of space-enabled services can include denial of service to communications satellites; loss of navigation capabilities for users of space services on the ground, in the air, and at sea; for military users of the GPS, loss of battlefield intelligence and inability to use precision-guided munitions; loss of timing fidelity that could lead to disruption in many networks that depend on tight synchronization with the GPS clocks; and degradation of weather forecasting and air traffic control due to loss of weather satellites. These consequences can be measured by a monetary amount representing the decision maker's willingness to pay to avoid them. The full risk description is thus a probability distribution of the losses per time unit. Their expected value, however, is an inadequate measure since it assumes risk neutrality. Therefore, expected disutility functions are used further to represent the value of an uncertain distribution of losses per time unit to a risk-averse decision maker.

The value of information of the system is quantified as the amount one is willing to pay (thus added to potential loss) to obtain the information {m} before making a tactical or strategic decision. The certain equivalent of a "lottery" is the sure thing that has the same value to the decision maker as the expected value of the lottery ("selling price") [7]. For a decision maker with constant risk aversion, the value of information is simply the difference of the certain equivalents of the "lotteries" that are faced in the decision to be made, with and without the information system accounting for error rates. It is on the basis of the improved value of information that the alternatives of different options to protect a satellite or to add to the monitoring systems are compared further.

3. COLLISION RISK MODEL AND BENEFITS OF ADDED SENSORS

The probabilistic model used here to assess the value of added sensors is structured as follows:
Step 1: Posterior probability of loss of a satellite given a combination of signals from the A and B monitoring systems
Step 2: Posterior probability of loss of a constellation conditional on such a message given that it takes a minimum number of satellite failures to cause the failure of a constellation (class of satellites)
Step 3: Tactical decisions to take countermeasures for a specific satellite based on the message, and value of information of the existing monitoring systems in those decisions
Steps 4 and 5: Risk reduction benefits (value of information) of increasing the main monitoring system (A) by one unit in an optimum location (that which protects most satellites with high values), versus increasing a number of other less accurate but less expensive sensors in system B for the same price as one sensor in A.
Step 6: Strategic decision: optimal allocation of added monitoring sensors, either one in A, or several in B, based on the minimization of expected disutilities, thus the maximization of increased value of information.

3.1 Notations

As mentioned above, we consider two monitoring systems (A the most accurate) and B (cheaper but not quite as accurate), each with a total number of sensors N_A and N_B. There are N_C monitored classes of satellites C_i (constellations) with N_i satellites S_{ik} in each constellation i. There are N_M classes of sensors M_j in A (index j for the class) with l sensors M_{jl} in each class j. Each class of sensors j in A includes N_j sensors. A binary function L describes the link between a class of sensors and the constellations that it observes in system A: L_{Aij}=1 if sensor class j monitors constellation i, 0 otherwise. Sensors in B indexed in j' ($M_{j'}$) are assumed to monitor all satellites; therefore: $L_{Bij'}$=1 for all i's and j'(s). Events of interest are a collision threat to each satellite ik (event X_{ik}) and to a whole constellation i (event X_i). A constellation fails (X_i) if the number of failures in that satellite class exceeds NF_i[‡]. The non-occurrence of events are noted by underlining the event's notation: $\underline{X}_{ik}$ and $\underline{X}_i$. The prior probability of X_{ik} per time unit (before any signal and correcting maneuvers) is noted x and is assumed to be the same for all satellites. Events X_{ik} are considered independent[§].

Note N_{Ai} the number of sensors in system A that monitor constellation i (and therefore satellite S_{ik}). N_B is the number of sensors in system B, all of which monitor any satellite S_{ik}. Therefore, all sensors in B belong to the same class, and are noted $M_{j'}$. Each sensor M_{jl} and $M_{j'}$ can send collision threat signals (noted in quotation marks) about each satellite to which it is connected ("X_{Ak}" and "X_{Bk}"), with probabilities of false positive, p(fp_A) and p(fp_B) respectively. Each sensor may also miss the threat and fail to issue a signal ("$\underline{X}_{Ak}$" or $\underline{X}_{Bk}$") with probabilities of a false negative, p(fn_A) and p(fn_B) respectively. These probabilities correspond to the following conditional probabilities:

‡ We note in the same way the threat and the failure but distinguish them in the computation.

§ It may not be the cases if they generate debris in a collision

p("X_{ak}"|X_k) is the probability of a true positive from A= 1-p("$\underline{X}_{ak}$"|X_k)= 1-p(fn_A).
p("X_{bk}"|X_k) is the probability of a true positive from B = 1-p("$\underline{X}_{bk}$"|X_k)=1- p(fn_B).

The messages received by the sensors are positive signals that a satellite (e.g., S_{ik}), is threatened. These positive signals include both true and false positives[**]. Each message, noted {n_{Ak}, n_{Bk}} represents a number of positive signals from A (n_{Ak}) and from B (n_{Bk}). The prior probability of a collision per time unit is assumed to be the same for all satellites and is noted x (note that in reality, it depends on its size and other factors). The posterior probability given a message is noted p(X_{ik}|{n_{Ak}, n_{Bk}}).

3.2 System structure

To recapitulate: the structure of the system considered here is thus formed of two sets of sensor classes, A and B. Set A is divided into classes of sensors M_j, each of which monitors a set of satellite constellations C_i indexed in i, formed of satellites C_{ik}. Set B sensors, Mj', monitor all satellites that it can detect. The links in the system are represented by the binary function Lij that indicates whether a particular set of sensors M_j in A monitors a particular constellation C_i, thus all its satellites $C_{ik.}$

3.3 Posterior probability of a satellite collision threat given sensor signals but before countermeasures

The posterior probability of X_{ik} given a message including n_{Aik} positive signals from A and n_{Bk} from B -before countermeasures- is obtained by Bayesian updating of prior x given the message {m}= {n_{Aik}, n_{Bik}} from all sensors in B, and the sensors in A that are observing satellite ik, i.e., for which L_{ij}=1.

- Numbers of sensors monitoring C_{ik}

From A: $N_{Ai}= \sum_{j=1}^{j=N_j} L_{ij}$. From B: $N_{Bi} = N_B$ i.e., all sensors in B. Eq. 1

- n_{Aik} and n_{Bik} are the numbers of positive signals from A and from B regarding satellites ik (i.e., so far, they do not include 0). They can be true or false positives. The numbers of signals (positive or not) about Sik, ntp_{Aik} include: the number of true positives and false positives from A, ntp_{Aik} and ntp_{Bik}; the number of true positives and false positives from B, ntp_{Bik} and nfp_{Bik}; $n_{Aik}= ntp_{Aik} + nfp_{Aik}$ and $n_{Bik}= ntp_{Bik} + nfp_{Bik}$ Therefore:

p({n_{Aik}}|X_{ik}) = p(ntp_{Aik}|X_{ik}) + p(nfp_{Aik}|X_{ik}) Eq. 2

p({n_{Bik}}|X_{ik}) = p(ntp_{Bik}|X_{ik}) + p(nfp_{Bik}|X_{ik}) Eq. 3

- The posterior probability of a threat given the message {m}={n_{Aik}, n_{Bik}} is derived by Bayesian updating of the prior x:

p(X_{ik}|{n_{Ak}, n_{Bk}}) = x . (p({n_{Aik}, n_{Bik}}|X_{ik}) / p({n_{Aik}, n_{Bik}}) Eq. 4

for all sensors in B, and for sensors M_{jl} in A that observe S_{ik}, i.e., such that L_{ij}=1.

- Negative signals in a time unit (no observation, i.e., $n_{Aik} = n_{Bik} = 0$) may be true negative or false negatives. False negatives may occur because the sensors cannot detect a piece of debris too small to be seen but large enough to cause damage to a satellite. In that case, the probability of collision remains at the level of prior x (in which it is included).

- Likelihoods are noted briefly as y_A, y_B, z_A, z_B to allow simplification of the posterior equation. The likelihood of the message given the threat is:

[**] One of the major threats to a satellite is a false negative, for example, a piece of debris large enough to do damage but too small to be seen given the present technologies[**], and the monitoring system as it exists now may not allow for protective maneuvers. The probabilities that it occurs are included in the priors. Additions of sensors as shown later, may have benefits in this case.

$p(\{n_{Aik}, n_{Bik}\}|X_{ik}) = p(\{n_{Aik}\}|X_{ik}) \times p(\{n_{Bik}\}|X_{ik})$ Eq. 5

for all sensors that observe ik as the messages are assumed to be independent conditional on the threat.

- The likelihood y_A of ntp_{Aik} (true positive messages from A) given a threat, $p(\{ntp_{Aik}\}|X_{ik})$, is the probability of the number of positive messages np_{Aik} given that there is a real threat of collision.

$$y_{A=} p(\{ntp_{Aik}\}|X_{ik}) = \binom{N_{Ai}}{ntp_{Aik}} p(tp_{Aik})^{ntp_{Aik}} . (1 - p(tp_{Aik}))^{N_{Ai} - ntp_{Aik}} \quad \text{Eq. 6}$$

in which p(tpA) = probability of a true positive (since the message is conditioned on X_{ik}) from all the N_{Ai} sensors in A that observe C_i .

- The likelihood y_B of the number ntp_{Bik} of true positive messages from B, $p(\{ntp_{Bik}\}|X_{ik})$ conditional on a real threat, is:

$$y_B = p(\{ntp_{Bik}\}|X_{ik}) = \binom{N_B}{ntp_{Bik}} p(tp_{Bik})^{ntp_{Bik}} (1 - p(tp_{Bik}))^{N_B - ntp_{Bik}} \quad \text{Eq. 7}$$

in which $p(tp_B)$ = probability of a true positive from each sensor in B.

- The likelihood z_A of the number nfp_{Aik} of positive messages from A given <u>no threat</u> is:

$$z_A = p(\{nfp_{Aik}\}|\underline{X}_{ik} = \binom{N_{Ai}}{nfp_{Aik}} p(fp_{Aik})^{nfp_{Aik}} . (1 - p(fp_{Aik}))^{N_{Ai} - nfp_{Aik}} \quad \text{Eq. 8}$$

- The likelihood z_B of the number nfp_{Bik} of positive messages from B given <u>no threat</u> is:

$$z_B = p(\{nfp_{Bik}\}|\underline{X}_{ik}) = \binom{N_{Bi}}{nfp_{Bik}} p(fp_{Bik})^{nfp_{Bik}} . (1 - p(fp_{Bik}))^{N_{Bi} - nfp_{Bik}} \quad \text{Eq. 9}$$

- The pre-posterior probability of the whole message {m} of a number of positive signals from sensor systems A and B, with or without a real threat is:

$$\begin{aligned} p(\{n_{Aik}, n_{Bik}\}) &= x . p(\{n_{Aik}, n_{Bik}\}|X_{ik}) + (1-x) . p(\{n_{Aik}, n_{Bik}\}|\underline{X}_{ik})) \quad \text{Eq. 10} \\ &= x. p(\{n_{Aik}\}|X_{ik}) \times p(\{n_{Bik}\}|X_{ik}) \\ &+ (1-x) [p(\{n_{Aik}\}|NX_{ik}) \times p(\{n_{Bik}\}|\underline{X}_{ik}) \\ &= x. p(ntp_{Aik}|X_{ik}). p(ntp_{Bik}|X_{ik}) \\ &+(1-x) (p(\{nfp_{Aik}\}|\underline{X}_{ik}) + p(\{nfp_{Bik}\}|\underline{X}_{ik})) \end{aligned}$$

=> The preposterior $p(\{n_{Aik}, n_{Bik}\}) = x. y_A . y_B + (1-x) z_A.z_B.$ Eq. 11

- The posterior probability of a collision threat for satellite X_{ik} given the prior x and the message $\{n_{Ak}, n_{Bk}\}$} is thus:

$$p'(X_{ik}) = p(X_{ik}|\{n_{Ak}, n_{Bk}\}) = \frac{x . y_A . y_B}{x . y_A . y_B + (1-x). z_A. z_B} \quad \text{Eq. 12}$$

3.4 Tactical decision of moving a sensor out of the way based on a set of positive signals

Once the message $\{m\}=\{n_{Ak}, n_{Bk}\}$ is received, the owner/operator of the satellite has the choice to take countermeasures to protect satellite S_{ik}, mostly by moving it out of the way to avoid a collision. That tactical decision is based on: the cost CM of moving a satellite (assuming that CM is the same for all satellites and that the maneuver does not cause an accident in itself); the posterior probability of a collision given the message; the loss incurred if the satellite is destroyed; and the risk aversion of the decision maker represented by his/her disutility for costs and losses.

The value of a satellite, in terms of revenue, is based on the revenues V_i derived from the operation of the constellation to which it belongs. That value is lost if a minimum number of satellites fail in the constellation. For simplicity, and since all satellites play an equal role in the sequence of potential failures, it is assumed here that the value of the constellation is equally shared among all its satellites. Therefore:

$V_{ik}= V_i/N_i$. Eq. 13

It is also assumed as mentioned earlier, that the risk aversion of the decision maker is constant over the range of consequences. Therefore the disutility of the operator $\underline{U}(.)$ can be represented as:

$\underline{U}(w) = 1 - e^{-\gamma w}$ Eq. 14

where γ = -$\underline{U}$"(w)/ $\underline{U}$'(w) is the risk aversion factor of the decision maker. It is here positive since marginal increases in the losses have a positive effect on the rate of increase of the disutility. The higher the curvature of the utility curve, the higher the level of risk aversion.

The tactical decision given the message {m} is thus to choose the option (to move or not the satellite for which the message was received) that minimizes the operator's expected disutility. If the decision is not to move, the expected disutility in the next time unit after receiving the message is $p'\underline{U}(V_i/N_i)$. Otherwise, after the maneuver, the probability of a collision per time unit returns to the prior x and the overall expected disutility for the next time unit is $x.\underline{U}\,(CM + V_i/N_i)$. The optimal choice is thus:

$\text{Min}\ [p'.\underline{U}(V_i/N_i)\ ;\ x.\underline{U}\,(CM + V_i/N_i)]$ Eq. 15

Note $\underline{U}$* the disutility of this optimum tactical choice for satellite ik. The probability of collision p*, in the next time unit, associated to that optimum is either the prior x (if the satellite is moved) or the posterior p' (if it is not moved) based on the signals received. Among all satellites for which a message was received, call α_i the proportion of satellites of constellation C_i that will be moved based on messages. α_i is based on (1) the posterior probability of failure of each of them, (2) the value of the constellation in general thus of each satellite in particular and (3) the risk aversion of the decision maker. Based on that decision for each specific case, one can assess the value of the countermeasures and the certain equivalents of the "lotteries" faced with or without the signals. Give the exponential form of the disutility function, the value of information of the monitoring system for that particular satellite is the difference of these certain equivalents.

3.5 Posterior probability of losing a constellation and risk of data losses. Value of information of the current monitoring system.

For a constellation C_i to fail, a minimum number (NF_i) of satellites in the constellation must fail. The nature and number of messages received (possibly none) allow computing a posterior probability of failure for each satellite X_{ik} in the constellation. Note nf_i the number of satellites lost to collisions in C_i before a threat to S_{ik} before taking countermeasures to protect it. The probability of failure of the constellation in the absence of signals, in the dangerous case where NF_i-1 satellites have already failed and where X_{ik} may be "the last straw", is:

$$\begin{aligned} p(X_i) &= p(nf_i = NF_i-1)\ .\ p(X_{ik}) \\ &= p(X_{ik})\ \text{Binomial}\ (N_i, x, NF_i-1)\ . \\ &= p(X_{ik}).\binom{N_i}{NF_i-1}\ x^{NF_i-1}\,.\,(1-x)^{N_i-NF_i+1} \end{aligned} \quad \text{Eq. 16}$$

Before the decision is made to move X_{ik} or not, the posterior probability of failure of the constellation C_i is thus the posterior probability of failure of satellite S_{ik} in the extreme case where the operator has received a message {nA, nB} of collision threat to S_{ik}, and where S_{ik} is the last satellite that keeps the constellation running.

=> $p'(X_i) = p'(X_{ik})$ Eq. 17

The immediate value of the information about the message {m} for the whole constellation is the difference of the certain equivalents of the lotteries faced, for that constellation, based on the move or not move (tactical) decision for the specific satellite X_{ik} about which the message was received.

CE(Move, given previous failure of NF_i -1 satellites and message about S_{ik}) => return to $p(X_{ik}) = x$)
CE(Move $S_{ik}\{nA_{ik}, nB_{ik}\} = \underline{U}^{-1}\{E\underline{U}$ failure lottery for X_i and cost of move}

$$= \underline{U}^{-1}[x.\underline{U}(V_i + CM)] \quad \text{Eq. 18}$$

CE (No move given previous failures of NFi -1 satellites and message of threat about S_{ik}), therefore $p(X_{ik})$ remains the posterior $p'(X_{ik})$.

$$= \underline{U}^{-1}[E\underline{U} \text{ failure lottery for } X_i]$$
$$= \underline{U}^{-1}[p'(X_{ik}) .\underline{U}(V_i)] \quad \text{Eq. 19}$$

The value of information in this particular case (waiting until the last satellite is threatened to take protective measures for the constellation i) is a reduction of the certain equivalent of the costs/losses with a move (if that is the best option):

$$VoI_i = \underline{U}^{-1}[p'(X_{ik}) .\underline{U}(V_i)] - \underline{U}^{-1}[x.\underline{U}(V_i + CM)] \quad \text{Eq. 20}$$

To simplify the computation, it is assumed here that the total value of information for all sensors can be limited to cases where a constellation i is protected by the move of the last satellite that is threatened, based on a message {m} after NF_i–1 satellites have failed[††]. In that case:

$$VoI = \sum_{i=1}^{1=N_c} \sum_{k=1}^{N_i} VoI_i. \quad \text{Eq. 21}$$

Obviously, in reality, most organizations may not wait until that threat materializes, and will probably choose to move some satellites earlier. In any case, the value of information of the whole sensor system is linked to the sequence of tactical decisions that will protect each constellation. One way to formulate the problem, which is beyond the scope of this paper, is to consider that sequence of tactical decisions for each constellation as a Markov decision process. The number of failures, with or without replacement at each time unit in a defined time horizon, should then be considered in each decision to protect a satellite S_{ik} based on the previous number of failures in the constellation i, on the future risks of losing that constellation, and on a new message [m} about S_{ik} even if it is not the last satellite standing in constellation i.

3.6 Increase in the value of information by adding one sensor in an optimal part of system A

For each satellite, adding a sensor XA in the part of the monitoring system that tracks it, modifies (1) the posterior probability of a collision based on a system of NA+1 sensors instead of NA and (2) the probability of a false negative, i.e., missing a collision threat. The first question is: which sensor system would provide the maximum value of information for the cost (CXA) given the set of satellites that it monitors?

For each sensor class j, consider the constellations C_i that it monitors (L_{ij}=1). The values of information as differences of certain equivalents across these constellations can be added to assess the value of information of the sensor group j for the constellations that it monitors. Adding a sensor changes the probabilities of error of the whole sensor system A, thus the posterior probability of X_{ik} given a message

[††] In a more conservative pattern (i.e., not waiting for the possible loss of the last satellite whose failure would cause that of the constellation), one could consider explicitly the proportion α_i of satellites that will be moved given messages {m} that include some positive signals during the overall time horizon. The derived posterior probability of failure for the whole constellation would then yield a value of information as a function of a decrease of the expected losses as well as the costs (and potential risks) of moving some satellites. Because it is based on differences of certain equivalents, that value of information could be added for the total number α_i.Ni of satellites that will be moved to protect the constellation, provided that in each case, the benefits justify the costs.

$\{n_A, n_B\}$, $p(X_{ik})$. That posterior probability becomes $p'(X_{XAik})$ as assessed in Equation 12. $p'(X_i)$ as shown in Equation 16 becomes p'_{XAi}. Therefore the value of information of adding one unit to the sensor group j is:

$$\text{VoIj} = \sum_{i=1}^{i=N_C} L_{ij}\, VoI_i(p'_{XAi}). \qquad \text{Eq. 22}$$

From this equation and the equations above, one can derive the value of information linked to the tactical decision to move or not satellite S_{ik} for any sensor group in A. Based on the posterior $p'(X_{ik})$, one can then compute the overall value of the information of each possible new sensor structure. Given that all constellations do not have the same value, if one sensor were added to A, it should it be located in the sensor group where it yields the highest value of information. For each sensor group j, and for the constellations i for which $L_{ij}=1$, the increase in the value of information of each sensors group is computed using Equations 1-21, changing NA to NA+1 and leaving NB unchanged. The result is a value of information VoI'_j, assuming an addition of one unit to each group j. The benefits in each constellation come from the change in the posterior probability of failure of each of its satellites given the message. One can then identify the sensor group j* in A that yields the maximum value of information VoI_{Aj*} for the whole monitoring system. The extra sensor should thus be added to the class j*. The cost is CXA and the benefit is:

$$\text{Ben (XA)} = VoI_{Aj*} - VoI \qquad \text{Eq. 23}$$

3.7 Risk reduction benefits (increase in the value of information) of adding several sensors in system B

Assume that the cost of an extra sensor in B, noted CXB, is a fraction of CXA and that one can get NXB extra sensors in B for the cost of one extra sensor in A. The added value of information is computed using again Equations 1-21 and changing NB to NB+NXB leaving A unchanged. The benefit of that addition is derived from the new posterior given the message $\{n_A, n_{B+NXB}\}$. As described above for A, that change and that of the value of information VoI_{XB} can be assessed based on the new probabilities of false positive and false negatives.

The cost is that increase of B by NXB units is CNXB = CXA
The benefits are: Ben (NB+NXB) = VoI_{XB} -VoI Eq. 24

3.8 Optimal strategic decision

Assume again that the preferences of the strategic decision maker can be represented by a convex exponential disutility curve as shown above, reflecting a constant risk aversion along the cost axis. The optimal strategy –adding 1 sensor to A, or XNB to B for the same price- is the option that maximizes the incremental value of information as shown in Equations 23 and 24.

4. CONCLUSIONS

When several sensors provide different messages, it may be tempting to simply use a voting rule or other basic heuristics. The problem is that one may then ignore the differences in the accuracy of the sensors. Also, some sensor groups monitor satellites in constellations of different values, i.e., those of the data that they provide. Upon receiving the message, a tactical decision has to be made to protect or not the unit at stake. Bayesian methods of integration of prior probabilities of failure, accuracy of the different sensors and potential losses and costs, allow supporting a rational decision at that stage. At the next level, one may consider the strategic decision to add sensors to the monitoring systems, either expensive, more accurate units, or a larger number of less costly but less accurate ones. One can make a rational choice by comparing their value of information as presented here, and by choosing the option that maximizes this net benefit. Both decisions are sensitive to the level of risk aversion of the decision makers (the satellite operators and the managers of the monitoring system). The Bayesian model presented here supports these two types of decision in a coherent way and can be extended to similar cases, for example in the medical field.

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