

# A Comprehensive Sensor Placement Determination in Condition Monitoring Process Using Combined Fault Detection, Fault Diagnosis and Risk Indexes

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**Abstract:** Increasing of maintenance cost is caused more technical interests on as-needed maintenance methods such as condition-based maintenance instead of scheduled maintenance. Selection of the location, type and number of sensors are important metrics of sensor network optimization. In this study, a novel stochastic approach is applied for determining optimal sensor locations. Three independent indexes are considered as decision-making parameters which reflect the efficiency of sensor network. First criterion is determined due to the uncertainty of sensor information which shows the ability of fault diagnosis of a sensor network. With about variation of environmental factors conditions (e.g. temperature) and their failure threshold characterization, system failure model is developed and analyzed by a proposed efficient Monte Carlo simulation. Statistical variance of sensor information in estimating of system state, the quantitative uncertainty measure of choice in this research, is estimated as the information value that each possible sensor placement scenario provides through sensor information. Second criterion is the reliability of sensors which reflects the fault detectability in sensor networks. A dynamic failure model is developed between sensors and their corresponding components. Then occurrence probabilities of top events are calculated for all placement scenarios. Risk of sensor failure is considered as third criterion which includes the consequence of sensor failure. In the next step, all scenarios are prioritized based on proposed criteria. Regarding to the results, it is concluded that considering proposed criteria independently misleads the decision maker about the optimal placement scenario. Accordingly, a combinatorial index is determined through Shannon Entropy theory which covers all criteria simultaneously. Finally all scenarios are prioritized based on proposed combinatorial index. As a case study, optimization of sensor placement is demonstrated on a typical steam turbine and results are discussed.

**Keywords:** Fault diagnosis, Information uncertainty, optimal sensor placement, Risk-based optimization, sensor reliability

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## 1. INTRODUCTION

Condition monitoring process requires efficient sensor network for health state estimation of complex systems. Fault detection, simultaneous fault distinction, uncertainty of sensor data and sensor reliability are main challenges in designation of monitoring process. Location, type and number of sensors are important metrics, affecting sensor network functionality effectiveness. There are three main available stochastic approaches for optimum determination of sensor location: i. Information uncertainty, ii. System reliability and iii. Risk of sensor failure. In the first category, placement of sensors is optimized based on sensor data uncertainty. The information contains different types of uncertainties justifying the probabilistic expression of sensor information in condition monitoring process. Jackson and Mosleh [1] presented a Bayesian approach for generating inference from multiple overlapping higher-level system data sets on component reliability parameters. Considering overlap data from different sensors in a system, optimization of sensor placement is studied as a case

study in this research according to less information uncertainty. Jackson and Modarres [2] prepared a review of how overlapping sensor data is analyzed in a Bayesian framework. Accordingly, a sensor placement optimization process has been formed for maximizing the information. Developed process is effective where sensors are expensive to install with various resource constraints (such as volume and weight), limiting their use. Prior information is used in proposed methodology to simulate evidence sets, which are then used to simulate posterior distribution of reliability metrics of interest. Information utility is derived from these posterior distributions, and an expected information utility is then attributed to sensor placement. Pourali and Mosleh [3] utilized a Bayesian Belief Network (BBN) –based sensor placement optimization methodology. Functional topology of the system, physical models of sensor information, and Bayesian inference techniques have been used in the approach along with the constraints.

In the second category, effect of sensor failure is studied on overall system failure. It is difficult to diagnose particular sensor faults. Therefore, different fault modes seen in sensors are investigated and faults are also simulated [4]. The fault models can then be used in simulated sensor fault scenarios to ensure that algorithms can distinguish between sensor faults and system faults. Accordingly, effects of failure modes are studied in detail and classified based on their criticality and occurrence probability [5]. A method is also proposed for taking corrective actions for eliminating the occurrence of various failure modes. Different techniques for sensor fault detection, disambiguation, and mitigation are also studied [6]. The research presents an expert system that uses a combination of object-oriented modelling, rules, and semantic networks to deal with the most common sensor faults, such as bias, drift, scaling, and dropout, as well as system faults. There is limited research on reliability of sensors and its effect on overall reliability of system health estimations while failure of sensors is a common event during the main system lifetime. Sensors are also considered as components in system failure model development [7]. Then a dynamic model is developed for evaluating the effect of time dependencies of sensors as well as components failure. Because of that, PAND gate is added to the system failure model between all sensors and their corresponding components to develop the failure model of each sensor placement scenario.

In the third category, system failure costs are considered in optimal sensor placement process. The optimal location of sensors can be identified based on the expected cost specifications [8]. In the proposed approach, minimal expected diagnosis cost is considered as the objective function for the sensor optimization with a fixed number of sensors. However, the consequences were neglected for system failures due to sensor failure. Economic consequences of sensor failure are irrecoverable in respect to the monitoring system as well as system failure [9].

According to the literature, sensor placement prioritization results were different due to considering each criterion separately. In the present study, the main motivation is dedicated to define a comprehensive criterion considering all proposed indexes simultaneously. First, Information uncertainty index is developed based on deviation of sensor information in estimation of system state. This index is considered as the ability of sensor network in diagnosing system faults. Then, sensor reliability index is determined based on the probability of top event in system dynamic fault tree analysis. Proposed index is considered as the ability of sensor network in detecting system faults. In the third step, a risk-based criterion is defined in dealing with sensor failure occurrence as well as the loss consequences of operation and maintenance in determination of system optimal sensor placement arrangement for both false-alarm and missed-alarm of sensors. Finally, using Shannon Entropy Theory, a combinatorial index is determined through considering all three proposed indexes simultaneously. In section 2, diagnosis ability of sensor network is discussed through determination of information uncertainty index. In this section, failure-causes of the system are determined through developing the system failure model. Accordingly, state vectors are determined based on different occurrence combination of failure-causes. Potential places of sensors are also identified in this section. Then sensor placement scenarios are ranked based on obtained uncertainty index. In section 3, detection ability of sensor network is illustrated through dynamic functional modelling of system failure. In section 4, sensor failure and corresponding mechanisms are studied for the dominant failure modes of the system. Occurrence probabilities are estimated for sensor missed-alarm and false-alarm in this section. Then risk model is developed for sensor failure. Reliability of sensors and magnitude of losses are obtained due to both sensor failure types. In section 5, a combinatorial index is determined

based on all three proposed indexes. In section 6, a case study is discussed on optimal sensor placement determination of a steam turbine. The results are discussed with the concluding remarks provided in section 7.

## 2. Diagnosis Ability of a Sensor Network

Considering sensor data, collected in different states of system's components, information of system health is usually obtained in the process of condition monitoring. The information inevitably contains different sources of uncertainties justifying the probabilistic expression of sensor information. Considering this fact, the most efficient sensor network in diagnosing system faults is determined as the configuration with minimum sensor information uncertainty.

First, the system components and their failure causes are identified. This is achieved by utilization of system Failure Modes and Effects Analysis (FMEA). The failure model of the system is developed through Fault Tree Analysis (FTA) to configure the logical relation of system components and related failure causes. Then, all combinations of failure causes are developed to form the state vectors []. The  $i^{\text{th}}$  state vector is written as eqn. (1):

$$SV_i = \{a_1, a_2, \dots, a_j, \dots, a_n\} \quad (1)$$

where “n” represents the quantity of failure causes in the system model, “ $a_i$ ” illustrates a binary variable equal to 1 if the  $i^{\text{th}}$  failure cause was occurred and equal to 0 in opposite direction. As an example, state vectors of a system with three failure causes are determined in **Table 1**.  $SV_1$  represents occurrence of all failure causes whereas  $SV_8$  represents a state in which none of failure causes were occurred.

**Table 1: State Vectors of a Typical System**

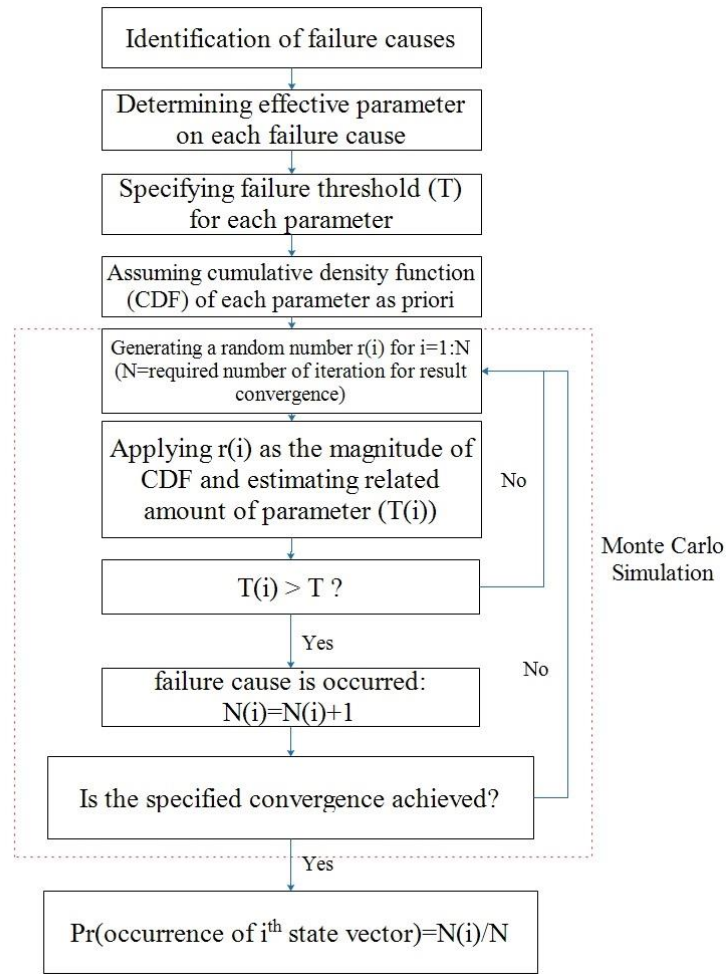
Failure causes	SV <sub>1</sub>	SV <sub>2</sub>	SV <sub>3</sub>	SV <sub>4</sub>	SV <sub>5</sub>	SV <sub>6</sub>	SV <sub>7</sub>	SV <sub>8</sub>
Failure cause 1	1	1	1	0	1	0	0	0
Failure cause 2	1	1	0	1	0	1	0	0
Failure cause 3	1	0	1	1	0	0	1	0

Occurrence probability is calculated for each state vector through eqn. (2) as:

$$\Pr(SV_i) = \prod_{j=1}^n [(P_j)^{a_j} (1 - P_j)^{(1-a_j)}] \quad (2)$$

where “ $P_j$ ” represents the occurrence probability of  $j^{\text{th}}$  failure-cause. Effective variables such as temperature, humidity, vibration are determined on each failure-cause after identifying failure-causes of the system like overheating, severe humidity, aging following with failure thresholds and probability distribution determination for proposed variables. The failure is occurred when the variable exceeds specified preset threshold. Regarding to proposed assumption, degradation process on system variable was neglected in this study. Monte Carlo simulation, as shown in, **Figure 1** is utilized for estimation of occurrence probability for each failure-cause. A state vector is then generated in each step of proposed simulation based on occurrence of failure-causes. Eventually, each state vector occurrence quantity is proportioned by total number of iteration to estimate occurrence probability of proposed state vector.

**Figure 1: The Algorithm for Estimating Occurrence Probability of Each State Vector**



Potential feasible places of sensors are selected as initial locations to determine sensor placement scenarios in the determination algorithm. In this methodology, potential places of sensors are determined due to Reliability Importance of failure causes [10].

According to the previous step about state vectors, information vectors are determined for sensors in this step. Each information vector contains binary arrays; equal to 1 if the related sensor is activated and equal to 0, otherwise. Considering “p” sensors for each scenario, there are  $2^p$  information vectors. As an example, the information vectors are shown for a scenario with 3 sensors in **Table 2**.

**Table 2: Information Vectors for a Typical System**

Information vector	IV1	IV2	IV3	IV4	IV5	IV6	IV7	IV8
sensor 1	1	1	1	0	1	0	0	0
sensor 2	1	1	0	1	0	1	0	0
sensor 3	1	0	1	1	0	0	1	0

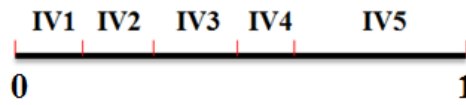
Occurrence probability of state vectors is estimated for each information vector. It is required to identify the state vectors activating the specific information vector. Then by summation of probabilities of proposed state vectors, the occurrence probability of  $s^{\text{th}}$  information vector is calculated as eqn. (3):

$$\Pr(IV(r)) = \sum_{i=1}^m \Pr(SV(i)) \cdot x_{ir} \quad (3)$$

where  $x_{ir} = \begin{cases} 1 & \text{If } SV(i) \text{ activates } IV(r) \\ 0 & \text{If } SV(i) \text{ doesn't activate } IV(r) \end{cases}$

Considering the probability of each information vector, the updated occurrence probability is estimated for each failure-cause using Monte Carlo simulation algorithm. In this approach, an interval with lower and upper bounds of “0” and “1” is divided based on probability of information vectors. Length of each interval is equal to the probability of related information vector as it is shown in **Figure 2**. In each step of simulation, a random number is sampled from a uniform distribution. If the random number falls in one of proposed intervals, it indicates the occurrence of corresponding information vector.

**Figure 2: Divided Interval with Lower and Upper Bounds 0 And 1 Based on Probability of IVs**



By occurrence of the specific information vector, associated state vectors are chosen. Finally, the probability of each failure-cause is estimated in each step by eqn. (4). If  $IV(r)$  is activated, then:

$$\Pr(C) = \sum_{i=1}^m (\Pr(SV(i)) \cdot x_{ir}) \cdot SV(i) \quad \text{with } C = \{C_1, C_2, \dots, C_n\} \quad (4)$$

where “C” represents the failure causes vector, containing “n” failure-causes. “s” is also illustrates the quantity of information vectors.

In the last step, information utility function is determined. The information utility function quantifies the state of knowledge of unknown parameters [9]. Information as the inverse of uncertainty is characterized on the magnitude of the variance of parameters. It is considered the most efficient when the smallest possible variance is achieved. In this research, probabilities of failure-causes occurrence are estimated through Monte Carlo simulation based on generating of information vector as discussed in step 5. Accordingly, a probability is assigned for each failure-cause occurrence in any iteration of the simulation process. By the definition of information uncertainty, variance of each failure-cause probability is determined in Monte Carlo simulation code developed in this research. It is clear that more dispersion on failure-causes occurrence probability results in less accuracy of system health condition estimation. Therefore, the information uncertainty index (UI) is determined as the summation of inverse variance of all failure causes probabilities as eqn. (5).

$$UI = \sum_{j=1}^n \frac{1}{\sigma_j^2} \quad (5)$$

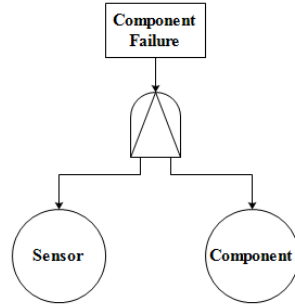
### 3. Detectability of a Sensor Network

In a condition monitoring process, sensor is mounted on the component to monitor a specific characteristic. In this case, if the proposed characteristic exceeds a determined threshold, sensor makes an alarm. This alarm will notify the operator and corrective actions are taken to prevent the occurrence of main failure. But if a sensor fails then the related failure will not be detected immediately. In condition-based maintenance, if a fault is appeared in the system component and sensor is working properly, the component will not be failed due to sensor alarm. So in this research, it is assumed that the failure of component is occurred only when the related sensor is failed before the component failure. So the sequence of failure between sensor and component is important in system failure estimation.

For applying time dependency on the system, a dynamic gate named Priority-AND is used. In this case, standard FTA of the system is replaced with DFTA. PAND gate is logically equivalent to an

AND gate where the input events must occur in a specific order. In this study, PAND gate is located between sensor and its corresponding component, shown in **Figure 3**.

**Figure 3: PAND Gate between Sensor and Related Component**



Applying this gate, dynamic-based model can be developed for all sensor placement scenarios. In the next step, applying a Monte Carlo-based algebraic method [7], the occurrence probability of top event is calculated for each scenario. Based on calculated probabilities, the optimal scenario will be selected.

#### 4. Sensor Failure Risk of Sensor Network

Sensor faults occur due to various reasons such as ageing, wear, manufacturing inefficiencies, incorrect calibration or mishandling and environmental conditions [11]. In addition, sensor failure are potentially categorized in two types of i) missed alarm and ii) false alarm. The former is occurred when the sensor does not alarm in case of fault occurrence and the latter is happened when sensor alarms while the operation state is normal.

Considering sensor missed alarm in condition monitoring process, probability of component functional failure (CFF) is calculated through eqn. (6):

$$\Pr(CFF) = \Pr(CF) \cdot \Pr(MA) \quad (6)$$

where  $\Pr(CF)$  illustrates the failure probability of the component and  $\Pr(MA)$  shows the occurrence probability of corresponding sensor missed-alarm. It shows that the functional failure depends on both component and related sensor failures. If there are several sensors in the system, affected by the proposed component failure, probability of system functional failure is calculated under condition monitoring process as eqn. (7):

$$\Pr(SFF) = \Pr(CF(I)) \cdot \prod_{l=1}^n (\Pr(MA(s)))^{d_{sl}} \quad (7)$$

where  $d_{sl}$  is a binary variable, the value is 1 if the failure of  $l^{th}$  component causes the  $s^{th}$  sensor to alarm and considered zero if it doesn't activate at all. In this case, sensor failure is simulated in complex systems with different sensors.

Sensor false alarm has no effect on the system functional failure because no fault has been occurred in the system. However, there are additional costs for unnecessary possible operation interruption, inspections and repairs, increasing the total cost. On the other hand, in case of the sensor missed alarm situation, economic consequences are not negligible for system failure and related maintenance costs. Taking into account all these factors, it is concluded that risk of the system due to sensor failure is a proper criterion for selecting of efficient sensor network because it covers both sensor failure and related consequences simultaneously. In the following section, risk assessment is described for the system for both sensors missed alarm and false alarm.

To assess the risk in case of sensor placement determination problem, the likelihood of different events and also related losses are considered for both missed alarm and false alarm cases.

In the missed alarm condition, instead of taking into account the risk of each failure cause separately, all failure causes are combined together through utilization of state vectors. The modified probability of system functional failure (SFF) is calculated for system due to missed alarm of sensor through eqn. (8):

$$\Pr(SFF) = \sum_{i=1}^m \Pr(SV(i)) \cdot \prod_{s=1}^p (\Pr(MA(s)))^{d_{is}} \quad (8)$$

where  $d_{is}$  is a binary variable, assumed 1 if the occurrence of  $i^{\text{th}}$  state vector affects the  $s^{\text{th}}$  sensor and zero otherwise.

Missed alarm condition causes substantial losses in the system. System failure cost is the main consequence of sensor failure, apportioned in maintenance program design. Cost of monitoring system is an addition in preventive maintenance process. This cost is determinant in sensor placement determination problem since it varies based on the quantity and type of sensors. Severity Factor is a proper index for reflecting the consequence of failure cause occurrence, obtained from FMEA. Accordingly, severity vector (SEV) is determined using FMEA table. Finally, risk of the sensor missed-alarm is determined as eqn. (9):

$$RoMA = \sum_{i=1}^m [\Pr(SV(i)) \cdot \prod_{s=1}^p (\Pr(MA(s)))^{d_{is}} \cdot ((SEV(i)) \cdot SV(i))] \quad (9)$$

Regarding to the eqn. (9), failure probabilities are multiplied for activated sensors by occurrence probability of  $i^{\text{th}}$  SV. The result is then multiplied by amount of effective severity, calculated through multiplying  $i^{\text{th}}$  SEV and SV. As an example, if there are three failure causes in each state vector, both SEV and SV are  $3 \times 1$  matrices. Therefore, multiplying SEV by transpose of SV in each step, a quantity is obtained which reflects the consequences of proposed SV.

In the false-alarm condition, only state vectors are applied which do not actuate all sensors. The reason is that if all sensors were actuated, there is not any sensor to make false alarm. Probability of false alarm occurrence is calculated through eqn. (10) as:

$$\Pr(FA) = \sum_{i=1}^m \Pr(SV^*(i)) \cdot \prod_{s=1}^p (\Pr(FA(s)))^{d'_{is}} \quad (10)$$

where  $d'_{is}$  is a binary variable, assumed 1 if the failure of  $i^{\text{th}}$  state vector doesn't affect the  $s^{\text{th}}$  sensor and zero otherwise. By augmenting the expert judgments by available experimental data, it is observed that probability of sensor false alarm is less than probability of sensor missed alarm in most cases.

False alarm condition results in economic losses due to unnecessary actions only and does not include any losses due to system failure. Thus, the main losses are inspection costs ( $C_{insp}$ ) and unnecessary repair costs ( $C_{rep}$ ). Finally, risk of the sensor false alarm is determined as eqn. (11):

$$RoFA = \left( \sum_{i=1}^m \Pr(SV^*(i)) \cdot \prod_{s=1}^p (\Pr(FA(s)))^{d'_{is}} \right) \cdot (C_{insp} + C_{rep}) \quad (11)$$

Eventually, considering both missed alarm and false alarm of sensors, risk of sensor failure is developed as eqn. (12):

$$RI = \sum_{i=1}^m [\Pr(SV(i)) \cdot \prod_{s=1}^p (\Pr(MA(s)))^{d_{is}} \cdot ((SEV(i)) \cdot SV(i))] + \left( \sum_{i=1}^m \Pr(SV^*(i)) \cdot \prod_{s=1}^p (\Pr(FA(s)))^{d'_{is}} \right) \cdot (C_{insp} + C_{rep}) \quad (12)$$

## 5. Determining a combinatorial index through Shannon Entropy theory

Finding the appropriate weight for each factor is the main point of a decision-making process. Shannon entropy method as an approach for determining weights of attributes in a decision-making process is developed based on the uncertainty of information [12]. Considering a sample MADM problem for “m” cases and “n” criteria, **Table 3** is developed.

**Table 3: Decision-Making Table**

Factors	Criterion 1	Criterion 2	...	Criterion w
Case 1	$a_{11}$	$a_{12}$	...	$a_{1w}$
Case 2	$a_{21}$	$a_{22}$	...	$a_{2w}$
Case 3	$a_{31}$	$a_{32}$	...	$a_{3w}$
...	...	...	...	...
Case v	$a_{v1}$	$a_{v2}$	...	$a_{vw}$

Since each criterion has reflects different aspect of cases, finding the appropriate weight for each criterion is the main points in MADM [13]. One of the objective weighting measures which has been proposed by researchers is the Shannon entropy concept [13]. Considering this fact, two main effective criteria (information uncertainty and risk of sensor failure) are applied and placement scenarios are prioritized based on them. Then efficient placement scenario is selected based on a weighted factor which is determined applying Shannon Entropy theory. Weight of each criterion in proposed theory is determined through eqn. (13):

$$W_j = \frac{dev_j}{\sum_{j=1}^w dev_j}; \forall j \quad (13)$$

where  $dev_j$  is calculated through eqn. (14):

$$dev_j = 1 - \left( -\frac{1}{\ln v} \left( \sum_{i=1}^v \frac{a_{ij}}{\sum_{i=1}^v a_{ij}} \ln \frac{a_{ij}}{\sum_{i=1}^v a_{ij}} \right) \right); \forall j \quad (14)$$

Finally, by maximum entropy deviation, Decision Making (DM) factor is developed using eqn. (15):

$$DM = \sum_{j=1}^n a_{ij} W_j \quad (15)$$

According to the results for DM, the efficient case will be selected. In the proposed approach, scenarios are considered as “cases” and diagnosis ability, detection ability and Risk of sensor network are supposed to be as criteria. Then regarding to Shannon Entropy theory, a weight factor is determined for each index. Finally, all scenarios are prioritized through obtained combinatorial criterion (DM).

## 6. Case Study: A Steam Turbine

Steam turbine belongs to a category of machines called turbo-machines, converting thermal energy of steam into mechanical energy. Steam turbines are expected as high-reliable machines operating continuously for long specified period. The main components of a typical steam turbine are diaphragms, rotor blades, bearings, rotor, seals, and casing. There are different types of monitoring available for steam turbine including steam quality, flow rate, vibration, lubricant/bearing conditions, rotor speed/load or power, auxiliary system operation and noise levels sensors [14].

Both information uncertainty-based and risk-based sensor placement determination results are studied in this application. First, failure causes of system components are identified. Failure modes and effects analysis of the steam turbine is developed and results are presented in **Table 4** [15].

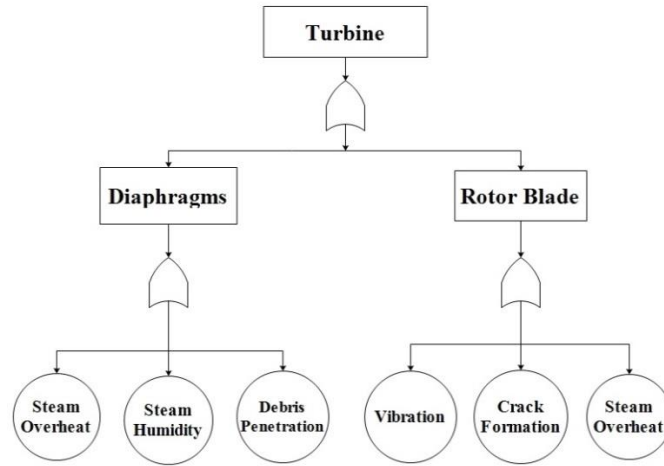


**Table 4: FMEA of a Typical Steam Turbine**

Component	Component Function	Potential failure modes/mechanisms	Occurrence	Potential causes of failure	Severity Index	Detection	Risk Priority Number (RPN)
Diaphragm	Convert thermal energy to kinetic energy by accelerating the steam	Erosion	4	Penetration of solid particles from or droplets from steam	4	3	48
		Scaling	2	Too dry steam	3	3	18
		Corrosion	1	Exposure to the corrosive substance in the steam	2	3	6
Rotor Blades	Convert kinetic energy to mechanical energy	Erosion	3	Penetration of solid particles from or droplets from steam	5	3	45
		Cracking	1	-Fatigue -Vibration	5	4	20
		Scaling	2	Too dry steam	3	2	12
Seals	Mechanical Sealing	Erosion	3	-Penetration of debris -Formation of droplets in the steam	2	2	12
		Corrosion	1	-Exposing to corrosive substances in the steam	1	3	3
		Rubbing	1	-Misalignment of rotor	2	2	4
Bearing	Support the rotor	Wear	1	-Aging	2	1	2
		Fracture formation	1	-Fatigue -Vibration	2	4	8
Rotor	Transfer mechanical energy to generator	Erosion	3	-Penetration of debris -Formation of droplets in the steam	2	3	18
		Corrosion	2	-Exposing to corrosive substances in the steam	2	3	12
		Misalignment of rotor	1	-Generator supports are skewed -Turbine supports are skewed	4	2	8
		Fatigue	1	-Aging	2	4	8
Casing	Protects the rotor and forms the steam path	Erosion	4	-Penetration of debris -Formation of droplets in the steam	2	3	24
		Scaling	3	-Too dry steam -High amount of substances in the steam	2	3	18
		Corrosion	2	-Exposing to corrosive substances in the steam	1	3	6

Comparing the risk priority number (RPN) of different failure causes in FMEA table, it is identified that diaphragms and rotor blades are high-risk components in the steam turbine. Therefore, their health monitoring has a higher priority. A simplified fault tree model is developed as **Figure 4** to clarify failure model of the steam turbine for these two components.

**Figure 4: Simplified Fault Tree Model of the Steam Turbine**



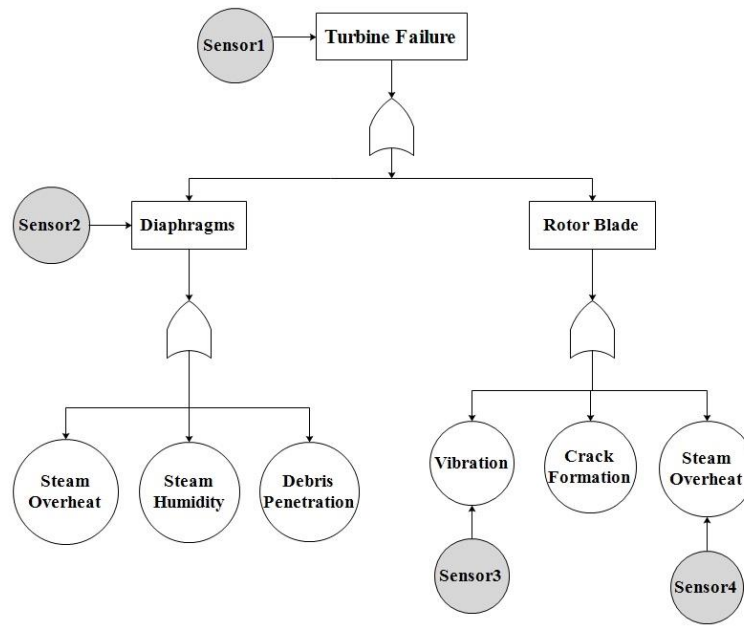
Steam temperature, relative humidity percent, percent of debris, vibration amplitude and crack size are selected as effective variables for simulating steam turbine failure. Due to limited information about proposed variables, expert judgment is augmented with the available data for better estimations of their probability density functions. Results are represented in **Table 5**. Operational and environmental conditions of the steam turbine variables were studied and related data about steam temperature, steam humidity, debris percent, vibration amplitude and crack size were estimated through both expert judgment and literature ([9]). Accurate results are achieved through collecting more precise information about system state.

**Table 5: Occurrence Probabilities of Failure Causes**

Failure parameters	Distribution	Threshold
Steam temperature (°C)	Normal(400,50)	600
Debris percent	Normal(2,0.5)	5
Vibration amplitude (μm)	Normal(10,1)	15
Humidity percent	Normal(50,5)	60
Crack size(mm)	Normal(0.5,0.1)	1

According to the Reliability Importance of failure causes, two sensors are mounted on the system to monitor steam temperature, and rotor blade vibration [9]. In addition, performances of diaphragm and turbine are monitored by other independent sensors [9]. Potential places of sensors are indicated in **Figure 5**.

**Figure 5: Potential Places of Sensors Demonstrated in the Fault Tree**



Types of sensors are selected based on their potential places in **Error! Reference source not found.**. Typical failure rate of each sensor is obtained using a generic database [16]. With limitations on sensors failure data, the constant failure rates are assumed of sensors in this study. Failure probabilities are calculated for sensors for life time of 4000 hours with the results presented in **Table 6**.

**Table 6: Types and Failure Rates of Sensors**

Sensor number	Sensor type	Failure rate (per 10 <sup>6</sup> hours)	Failure probability for 4000 hours
1	Tachometer	80	0.274
2	Wireless accelerometer	4	0.016
3	Accelerometer	174	0.5
4	Thermometer	1	0.004

According to reported failure database in reference [16], it is not specified there that which failure type (missed-alarm or false-alarm) is published. To correctly account for this issue, the probability of sensor false-alarm is determined as 25% of database reported sensor missed-alarm probability [17]. Results are shown in **Table 7**.

**Table 7: Probability of Sensor False Alarm**

Sensor number	Sensor type	False alarm probability for 4000 hours
1	Tachometer	0.07
2	Wireless accelerometer	0.004
3	Accelerometer	0.125
4	Thermometer	0.001

Determination of sensor placement is applicable in the case where quantity of the sensors is less than the potential places. Based on four potential locations for sensor locations in this study, it is assumed here that only three sensors are allowed to use in the given process. All possible scenarios are shown in **Table 8**.

**Table 8: Sensor Placement Scenarios**

Scenario number	Sensor number
Scenario 1	Sensor1, Sensor2, Sensor3
Scenario 2	Sensor1, Sensor2, Sensor4
Scenario 3	Sensor1, Sensor3, Sensor4
Scenario 4	Sensor2, Sensor3, Sensor4

According to eqn. (2), probabilities of state vectors are calculated. Results are shown in **Table 9**.

**Table 9: State Vectors of Steam Turbine**

Failure causes	SV1	SV2	SV3	SV4	SV5	SV6	SV7	SV8	SV9	SV10	SV11	SV12	SV13	SV14	SV15	SV16	SV17	SV18	SV19	SV20	SV21	SV22	SV23	SV24	SV25	SV26	SV27	SV28	SV29	SV30	SV31	SV32
Overheat	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	1	1	0	1	1	0
Humidity	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1	1	1	0	1	0
Debris	1	0	0	1	0	0	0	1	0	0	1	0	0	1	1	0	1	0	0	1	1	0	1	1	0	1	1	1	1	1	0	0
Vibration	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	1	1	0
Crack	1	0	0	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	1	0	1	1	0	1	1	1	0	1	1	1	1	0
Occurrence Probability	1e-8	0.0011	0.1511	0.0187	0.0011	0.0185	2.23e-4	3.6e-5	3e-6	2.5e-5	0.0035	1.88e-4	0.0035	2.4e-5	4.45e-4	2e-5	6e-6	1e-8	5e-6	1e-8	1e-6	1e-8	3e-6	9.2e-5	8e-6	1e-8	1e-8	1e-8	1e-8	1e-8	1e-8	0.8016

For each sensor placement scenario, probabilities are estimated for system information vectors using system state vectors through eqn. (3). Regarding to the arrangement of sensors in the system, some information vectors do not occur [10]. All possible information vectors are presented with their probabilities shown in **Table 10**.

**Table 10: Information Vectors of the First Scenario**

Sensor number	IV1	IV2	IV3	IV4	IV5
1	1	1	1	1	0
2	1	0	1	0	0
3	1	1	0	0	0
Probability	4.82e-6	0.001	0.1785	0.0186	0.8016

In this step, occurrence probabilities are calculated for all failure-causes. By applying eqn. (9), information uncertainty index is calculated for each scenario. Finally, the most efficient scenario is selected the one with higher value of the proposed uncertainty index. Results are shown in **Table 11**.

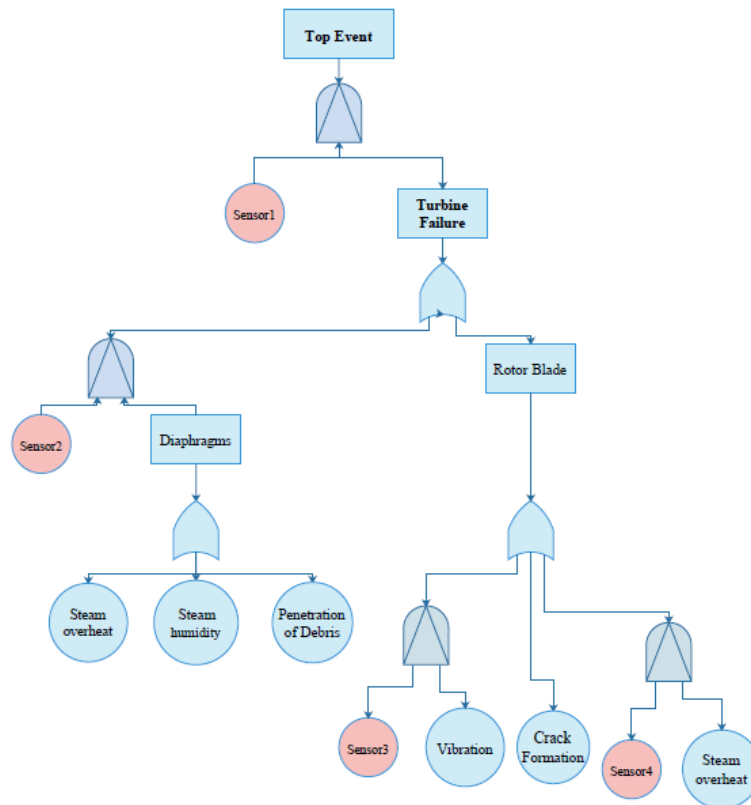
**Table 11: Information Uncertainty Index of All Scenarios**

Scenario Number	Information Uncertainty Index
Scenario 1	6.9e+08
Scenario 2	4.26e+08
Scenario 3	7.83e+08
Scenario 4	3.9e+08

**Table 11** shows the third scenario with the highest value for the uncertainty index, considered as the most efficient placement scenario. The fourth scenario is the worst scenario due to absence of sensor #1. In fourth scenario, occurrence of “crack formation” is not detected with the results reflected in the **Table 11**.

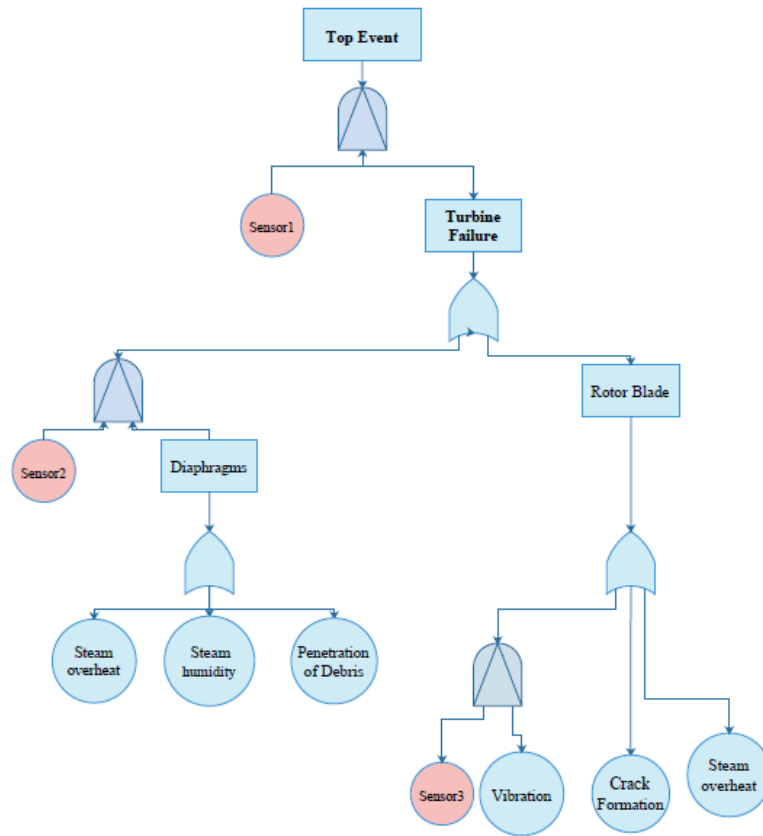
Applying PAND gate between sensor and related component, dynamic model of the system failure is developed. DFT of the system with all potential places of sensors is shown in **Figure 6**.

**Figure 6: Dynamic-Based Model of Sensor and Related Component**



As an example, dynamic failure model of the first scenario is developed and shown in **Figure 7**. Vibrations of rotor blade, functionality of diaphragm and turbine operation are monitored in this scenario. Also there are sensors in all three level of the system.

**Figure 7: Functional Model of First Scenario**



Using algebraic method for DFT calculation, cut sequences of the developed model are extracted as below:

- CS1: SO.(S1 ◀ SO)
- CS2: CF.(S1 ◀ CF)
- CS3: V.(S3 ◀ V).(S1 ◀ V)
- CS4: PD.(S2 ◀ PD).(S1 ◀ PD)
- CS5: SH.(S2 ◀ SH).(S1 ◀ SH)

Where “SO.(S1 ◀ SO)” means that steam overhear has been occurred and sensor #1 was failed before steam overhear. Other sequences are interpreted similar to this example. The simulation is done for 4000 hours and results are presented in Table 12.

**Table 12: Occurrence Probability of Sequential Events**

Sequential event	Probability of occurrence (for 4000 hours)
S1 ◀ CF	0.0075
S1 ◀ PD	0.051
S1 ◀ SH	0.05
S1 ◀ SO	0.015
S1 ◀ V	0.0067
S2 ◀ PD	0.0028
S2 ◀ SH	0.0026
S2 ◀ SO	0.00076
S3 ◀ PD	0.1
S3 ◀ V	0.014
S4 ◀ SO	0.00018
S4 ◀ V	0.00016

Table 5 illustrates that the probability of sensor failure before failure of component is extremely less than sensor failure probability. When results of Table 5 are used in equation 2, the probability of each cut sequence is affected significantly. Failure probability is calculated for top event of the first scenario through:

$$\Pr(\text{TopEvent}) = \Pr(\text{CS1}) + \Pr(\text{CS2}) + \dots + \Pr(\text{CS5}) - \Pr(\text{CS1})\Pr(\text{CS2}) - \Pr(\text{CS1})\Pr(\text{CS3}) - \Pr(\text{CS4})\Pr(\text{CS5}) + \Pr(\text{CS1})\Pr(\text{CS2})\Pr(\text{CS3}) + \dots = 0.002$$

Described process is applied on the remaining scenarios in **Error! Reference source not found.** Final results are illustrated in **Table 13**.

**Table 13: Results of Top Event Probability for Different Scenarios**

Scenario Number	Probability of top event
Scenario 1	0.002
Scenario 2	0.00088
Scenario 3	0.0398
Scenario 4	0.0586

**Table 13** shows the second scenario with the lowest value for the probability of top event, considered as the most efficient placement scenario.

Risk of sensor failure as the other criterion for sensor placement determination is calculated for each scenario according to eqn. (12). First, occurrence probabilities are estimated for state vectors. In next step, affected sensors are identified for each state vector. As an example, affected sensors are shown for all state vectors in first scenario in **Table 14**.

**Table 14: Affected Sensors of State Vectors in First Scenario**

State vectors	Affected sensors
SV1, SV9, SV12, SV14, SV18, SV20, SV22, SV23, SV25, SV26, SV27, SV29, SV30, SV31	Sensor1, Sensor2, Sensor3
SV5, SV16	Sensor1, Sensor3
SV2, SV3, SV4, SV7, SV8, SV10, SV11, SV13, SV15, SV17, SV19, SV21, SV24, SV28	Sensor1, Sensor2
SV6	Sensor1

As shown in **Figure 7**, occurrence of vibration and at least one of either steam overhear, steam humidity or penetration of debris result in all sensor activation in the first scenario. State vectors actuate all three sensors in the first row of **Table 14** where the other scenarios actuate only one or two sensors. Using state vectors and failure probabilities of related sensors, probability of system functional failure is calculated for each sensor placement scenario.

As discussed before, it is required here to extract state vectors which do not actuate all sensors in order to calculate the probability of sensor false alarm. State vectors and related sensors, which may alarm incorrectly in the first scenario, are represented in **Table 15**.

**Table 15: Feasible False-Alarmed Sensors of All State Vectors in the First Scenario**

State vectors	Feasible false-alarmed sensors
SV32	Sensor1, Sensor2, Sensor3
SV6	Sensor2, Sensor3
SV5, SV16	Sensor2
SV2, SV3, SV4, SV7, SV8, SV10, SV11, SV13, SV15, SV17, SV19, SV21, SV24, SV28	Sensor3

Using probabilities of state vectors and also false alarm probability of related sensors, probability of sensor false alarm is calculated for each sensor placement scenario. Finally, risk of sensor failure is calculated in the system using both sensor failure probability and related monetary losses. SEV is determined [3 2 4 5 5] according to **Table 4**. Cost types are also estimated through reported data from the reference due to sensor false-alarm [18], shown in **Table 16**.

**Table 16: Costs Due to Sensor Failure**

Type of cost	Value (unit of loss)
Inspection costs ( $C_{insp}$ )	0.15
Unnecessary repair costs ( $C_{rep}$ ).	1.5

Applying severity indexes and also normalized costs from **Table 16**, the risk index is calculated for sensor failure by eqn. (16) for all scenarios. Results are illustrated in Table 17.

**Table 17: Risk Index of Sensor Failure for Each Scenario**

Scenario Number	Risk index of sensor failure
Scenario 1	0.17
Scenario 2	0.155
Scenario 3	0.33
Scenario 4	0.152

According to Table 17, the scenario 3 has the highest amount of risk index whereas the scenario 4 has the least amount. Comparing with the results of information uncertainty index, rank of scenarios is vice versa. The scenario 3 is in the first place due to information uncertainty whereas risk of sensor failure is the highest for this scenario.

Finally, the combinatorial criterion is calculated for different scenarios. The most efficient placement scenario is selected through higher information uncertainty index, lower top event probability and lower risk index. However, in Shannon Entropy theory, both criteria must have the same trend. Accordingly, the inverse of information uncertainty is considered. Applying eqn. 17 and eqn. 19, DM is calculated for each scenario. Results are shown in **Table 18**.



**Table 18: DM Index for Different Scenarios**

Scenario	Normalized inverse information uncertainty	Normalized top event probability	Normalized risk	DM
1	0.65	0.0341	0.5151	0.1117
2	1	0.015	0.4697	0.1156
3	0.546	0.6791	1	0.694
4	0.408	1	0.4606	0.9195

According to **Table 18**, it is concluded that the rank of scenarios is completely different based on DM factor. The first scenario is the most efficient placement scenario regarding to the DM-based determination which is in the third place through both information uncertainty and risk indexes. It verifies that the most efficient criterion must be diagnosed at first and then the decision-making process applied for sensor placement determination.

### 3. Conclusion and Discussion

In this research, reliabilities of sensors and sensor failure consequences are studied as well as considering the uncertainty of sensor information to select the efficient places for sensors. Sensors are added to the failure model of the system as the system's components. Dynamic failure model is applied on functional model of system which reflects the nature of condition-based maintenance properly. Malfunction of sensors are categorized in missed-alarm and false-alarm types. Occurrence probability are estimated for proposed malfunctions and included in the system's failure model. Economical loss consequences are obtained due to failure of sensors and accordingly, risk index is determined based on occurrence uncertainty, sensor reliability and related losses. Eventually, placement scenarios are ranked and the most efficient scenario is selected.

The main achievement of this research is evaluation of mutual effect of information uncertainty, sensor reliability and risk of sensor failure in selecting efficient places of sensors. It is concluded that the results of sensor placement prioritization can be independent based on different criteria. It means that ignoring each of proposed criteria causes to select improper location for sensors.

Considering a steam turbine as a case study, the effect of proposed indexes is studied separately. According to the low variation of indexes, DM factor is calculated through simultaneous consideration of them due to Shannon Entropy theory. Finally, it is concluded that the DM-based ranking of scenarios is completely different and ignoring each of proposed criteria causes to select improper location for sensors.

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